Workshop 10, Week 10

Please follow the instructions of your supervisor regarding timing of these problems.

Math Review

1. ∗ Find the angle between the two vectors $\mathbf{a} = (1,0,2)$ and $\mathbf{b} = (2,2,2)$.

2. ∗ Find a unit vector orthogonal to both $\mathbf{a} = (1,0,2)$ and $\mathbf{b} = (2,2,2)$.

Physics Problems

3. ∗ Calculate the total mass of a 1m bar (cross section 2×2 cm^2) of an unevenly mixed alloy, where the density changes linearly from 10 kg/m^3 at one end to 20 kg/m^3 at the other.

4. The moment of inertia of an object, consisting of a number of point masses, relative to an axis is defined as $\mathcal{I} = \sum_i m_i r_i^2$, where r_i is the perpendicular distance of the point i to the axis.
(i) Generalise this expression to a volume of revolution (where the mass distribution is continuous) assuming a constant density ρ.
(ii) We construct a top by rotating the curve $y = 3/2 - x^2$, $-1 \leq x \leq 0$, $y = 3/2 - 3/2x$, $0 \leq x \leq 1$, around the x axis. Calculate its mass and moment of inertia around the x-axis.

5. We look at the electric field around a charged, infinitely long, insulating wire. Assume that the charge density is given by σ. Find the electric field (magnitude and direction) at a point a distance R from the wire
(i) Through the addition of all the electric fields (field due to a charge q: $\mathbf{E} = -kq\mathbf{r}/r^3$).
(ii) Through the calculation of the total potential ($V = kq/r$).

Maths Practice

6. Evaluate the following integrals

(i) ∗ $\displaystyle\int \frac{1}{1-5x+6x^2}\,dx$, (ii) $\displaystyle\int \frac{1}{u^2(a+bu)}\,du$,

(iii) ∗ $\displaystyle\int_0^1 \frac{dx}{1-2x+2x^2}$, (iv) $\displaystyle\int_{-1}^1 \frac{x^2+3x+1}{(x+2)^2(x+3)^2}\,dx$.

7. Evaluate the following integrals:

(i) ∗ $\displaystyle\int \frac{dx}{\sqrt{7+4x-4x^2}}$, (ii) $\displaystyle\int \frac{\sqrt{x-1}}{x}\,dx$,

(iii) ∗ $\displaystyle\int \frac{x}{\sqrt{1+2x-x^2}}\,dx$.

8. Find the areas between the curves. In each case make a sketch of the region.

(i) ∗ $y = x^3$; $y = x^2$.

(ii) $y = \frac{4}{x^2}$; $y = 5 - x^2$ $(x > 0)$.

(iii) Below $y = e^{-x}\sin(x)$ and above $y = 0$ from $x = 0$ to $x = \pi$.

(iv) Inside both of the circles $x^2 + y^2 = 1$ and $x^2 + (y-1)^2 = 1$.

9. Find the volumes of revolution when the region R is rotated about the x-axis:

(i) ∗ R is bounded by $y = x(2-x)$ and $y = 0$ between $x = 0$ and $x = 2$.

(ii) R is bounded by $y = 1/(1+x^2)$, $y = 2$, $x = 0$ and $x = 1$.

(iii) R is bounded by $y = e^{-x}$ and $y = 0$ and lies to the right of $x = 0$.

Reading for next week: Chapter 14 (Differentiating simple functions)