Workshop 4, Week 4

Please follow the instructions of your supervisor regarding timing of these problems.

Maths Review

1. Sketch the polar curves
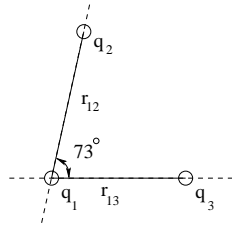
   (i) $*$ $r = 2\theta$,   (ii) $*$ $r = 1/(1 - \cos(\theta))$,

Physics Problems

2. $*$ A particle of mass 1g resides at the point (1,1,3), a particle of mass 2g is at (1,4,5) and one of 5g at (-2,-1,0). Find the position vector of the centre of mass.

3. $*$ Three charges, $q_1 = 4.0\ \mu C$, $q_2 = -6.0\ \mu C$, $q_3 = -5.0\ \mu C$, are placed in the xy plane.

   The distance between charges 1 and 2 is $r_{12} = 0.10$ m, $r_{13} = 0.15$ m, and the angle between r_{12} and r_{13} is $73°$. Find the force (size and direction) on charge 1. (Use $k = 1/(4\pi\epsilon_0) = 9 \times 10^9 \text{Nm}^2/\text{C}^2$.)

4. $*$ A cube has sides of length 1m which are parallel to the unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$. A force $\mathbf{F}$ of size 2 N acts parallel to the body-diagonal AD of the cube.
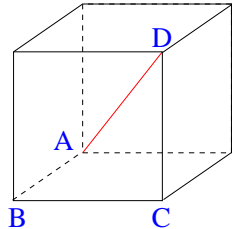
   Show that

   $$\mathbf{F} = \frac{2}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k}).$$

   Find the work done by $\mathbf{F}$ as it moves from A to D along the paths:
   (i) AD (ii) AB then BD (iii) AB then BC then CD.

5. A truck of mass 4000 kg is parked on a slope that makes an angle of $\pi/4$ with the horizontal. Find
   (i) The force exerted by the road.
   (ii) The handbrake is released. Find the acceleration of the truck.
   (iii) A bomb is exploded under the truck. Calculate the force required to give the truck an acceleration of g perpendicular to the road.

6. A small car of mass 950 kg travels eastwards, and collides with a car of mass 1900 kg traveling to the north. After the collision the wreckage sticks together and slides with a speed of 16.0 m/s in a direction $24°$ east of north. Calculate the speed of each vehicle before the collision (ignore friction).

Math Practice

7. $*$ Let $\mathbf{a} = (1, 2, -3)$ and $\mathbf{b} = (2, 0, 1)$. Find:

(i)	$\mathbf{a} + \mathbf{b}$,	(ii)	$\mathbf{a} - \mathbf{b}$,
(iii)	The length of $\mathbf{a}$,	(iv)	The length of $\mathbf{b}$,
(v)	$\mathbf{a} \cdot \mathbf{b}$,	(vi)	The angle between $\mathbf{a}$ and $\mathbf{b}$.

8. Given $\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$, $\mathbf{c} = \mathbf{i} - \mathbf{j}$, find

   (i) $\mathbf{a} + \mathbf{b} + \mathbf{c}$,
   (ii) $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$,
   (iii) $(\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} - \mathbf{c})$,
   (iv)   Find the magnitudes of $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$,
   (v)   and the angle between $\mathbf{a}$ and $\mathbf{b}$.

Reading for next week: **Chapter 5** (Trigonometric functions)