Workshop 7, Week 7

Please follow the instructions of your supervisor regarding timing of these problems.

Math Practice

1. Solve the following for x

   (i) $*$ $\quad -4\exp(x/4) + \exp(x/2) = -4,$

   (ii) $*$ $\quad \ln(1/(1+x^{1/2})) - \ln(y^{1/3}) = 2.$

Physics Problems

2. A planet of mass m moves under the influence of the gravitational force of the sun (mass M), $\mathbf{F} = -\frac{GmM}{r^2}\mathbf{r}$.
   (i) $*$ Show that the angular momentum, $\mathbf{L} = m\mathbf{r} \times \mathbf{v}$, is constant in time.
   Hint: Note that $\mathbf{r} = \mathbf{r}(t)$, and express the acceleration in terms of $\mathbf{r}$.
   (ii) **[Hard!]** Show that the motion takes place in a plane that contains the origin.
   Hint: Find the vector equation of such a plane!

3. $*$ A closed cylindrical container is to be constructed with volume 16π m^3, but to avoid using too much metal, the total surface area must be minimal. What are the dimensions of the cylinder?

4. Planck's radiation law say that the intensity of light emitted by a star (black body) of temperature T is

   $$I(\lambda) = \frac{2\pi hc^2}{\lambda^5(e^{hc/(\lambda kT)} - 1)}.$$

   Find the extremum of this curve as a function of T, and sketch the shape of the curve for a suitable temperature of 5000 K ($h = 6.6260755 \times 10^{-34}$ Js, $c = 299792458$ m/s, $k = 1.380658 \times 10^{-23}$ J K).

Maths Practice

5. $*$ Differentiate $(a^2 + \sin^2(\omega t))^{1/2}$ with respect to t.

6. For x from -4 to 4 find the smallest and largest values of $x^3 - 3x^2 - 9x - 10$. Sketch the curve.

7. Using logarithmic differentiation, find dy/dx for

   (i)$*$ $\quad y = (x+1)^2(x+2)^3(x+3)^4$, (ii) $\quad y = \sqrt{(3x^2+2)}\sqrt{(6x-7)}\,.$

8. By implicit differentiation, find dy/dx when

   (i)$*$ $\quad x^2 + y^2 + xy - 17 = 0$, (ii) $\quad x^2y^3 - x^2 + 3y - 3 = 0\,.$

   Also find d^2y/dx^2 for (i).

9. Find dy/dx and d^2y/dx^2 for $x = 5 + \sin t$; $\quad y = 3 - \cos t$ at $t = \pi/4$.

10. The function $\cosh(x)$ is defined as $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$, and $\sinh(x)$ as $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$. Find the derivative of the inverse function $\cosh^{-1}(x)$.

Reading for next week: Chapter 11 (Introducing scalars and vectors)