P201 Workshop 12, Week 12

Please follow the instructions of your supervisor regarding timing of these problems.

Revision: Mathematics II

1. Aolve $z^2 - 10z + 40 = 0$.

2. Calculate and sketch $f_r(x) := \mathrm{Re}[f(x)]$ and $f_i(x) := \mathrm{Im}[f(x)]$ for $f(x) = [x+i]^{-1}$ (real x).

3. Sketch the region of complex numbers in the complex plane (x–y plane) with $1 < |z| < 2$.

4. Calculate $f(t) \equiv \mathrm{Re}(e^{i\Omega t})$ where $\Omega = \omega + i\gamma$ and t, ω and γ are real.

5. Write down the infinite series which define Sine, Cosine, and the Exponential Function.

6. Calculate the derivative $\tanh'(x)$.

7. Solve $y'(x) = e^{px}, y(0) = 1, p \in R$.

8. Find the general solution of $y''(x) - y'(x) - 2y(x) = 0$. Now find the solution that fulfills $y(0) = 0$ and $y'(0) = 1$.

9. Write the general solution of $y''(x) + ay(x) = 0, a > 0$, written in terms of real functions.

10. Find the general solution of
(i) $y''(x) - 4y'(x) + 5y(x) = 0$;
(ii) $y''(x) + y'(x) + 12y(x) = 0$;
(iii) $y''(x) - ay(x) = 0, a > 0$.

11. Sketch $f(x) = x\sin(x)/(1 + x^2)$.

12. Calculate $1 + 2a + 4a^2 + 8a^3 + 16a^4 + ...$ Which condition must be fulfilled for this expression to converge?

13. Taylor expand the following around $x = 0$ (first 3 terms only):
(i) $f(x) = \sqrt{2+x}$; (ii) $f(x) = \sin(x)/x$.

14. Taylor expand the following to all orders around $x = 0$:
(i) $f(x) = e^{4+x^4}$; (ii) $f(x) = 1/(1 + x^6)$.

15. Find an approximation to $\sqrt{100-a}$, with $|a| < 10$, that gives at least two digits accuracy.

16. Calculate $\lim_{x\to 0} x^2/[\sin(ax^2)], a > 0$.

17. Calculate $\frac{\partial^2}{\partial y \partial x}\sin(x + x^2y^3)$.

18. Sketch the equipotential lines ($f = const$) and calculate the gradient and stationary points of (i) $f(x, y) = x^4 + y^4$; (ii) $f(x, y) = x^4 - y^4$.

19. Let $B = \begin{pmatrix} 1 & c \\ c & 0 \end{pmatrix}, \quad C = \begin{pmatrix} z & 1 \\ 1 & z^* \end{pmatrix}$. Now Calculate
(i) the determinant of B and C; (ii) BC; (iii) C^{-1};
(iv) The eigenvalues of B and C.