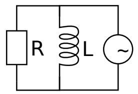
# P201 Workshop 2, Week 2

*Please follow the instructions of your supervisor regarding timing of these problems.*

## Physics

1. A quantum particle moving in a thin wire has a wave function $\Psi(x) = \dfrac{c}{x+i}$, where $x$ is the coordinate of the particle and $c$ a positive constant. Calculate and sketch the probability function $p(x) = \Psi^*(x)\Psi(x)$ of the particle ($p(x)dx$ describes the probability to find the particle in the small interval $dx$ around $x$). Where is the maximum of this probability?

2. Consider an AC circuit with external complex voltage $V(t) = V_0 e^{i\omega t}$ and complex current $I(t) = I_0 e^{i\omega t}$. The elements of a general circuit are resistors $R$, inductors $L$, and capacitors $C$. The admittance $Y_R$ of a resistor $R$ is defined as $Y_R = 1/R$, the admittance $Y_L$ of an inductor $L$ is defined as $Y_L = -i/(\omega L)$, and the admittance $Y_C$ of a capacitor $C$ is defined as $Y_C = i\omega C$.




(i) In a circuit with $R$ and $L$ parallel (see figure), the total admittance $Y$ is the sum of the two admittances, and the complex current amplitude $I_0$ is $I_0 = YV_0$ with $V_0$ being the complex voltage amplitude. Calculate the modulus $|I_0|$ of the current amplitude for this circuit.

(ii) The complex resistance of a capacitor $C$ is $Z_C = 1/Y_C = 1/i\omega C$. The total complex resistance $Z$ of a circuit with a resistor $R$ and a capacitor $C$ in series is $Z = R + Z_C$. Sketch the circuit and calculate the total complex current amplitude $I_0$ from $V_0 = ZI_0$, where $V_0$ is the total complex voltage amplitude. Then calculate the complex voltage drop $V_C$ at the capacitor and its modulus $|V_C|$. Identify a characteristic time-scale of this circuit.

## Math Practise

3. (i) Express the real and imaginary part of a complex number $z$, using $z$ and $z^*$.

(ii) Express sine and cosine in terms of the complex exponential function.

(iii) Sketch the following functions ($x$ real): $\ln(x)$, $\exp(-x)$, $\exp(-x^2)$, $1/(1+x^4)$.

(iv) Prove that $(1/z)^* = 1/z^*$ for any complex number $z$.

4. Calculate the following, expressing everything in the polar representation

(i) $\sqrt{i}$; (ii) $\sqrt{1+i}$; (iii) $(1+i)^2$; (iv) $(1+i)^4$.

5. (i) Why do inequalities like $z_1 < z_2$ make no sense for complex numbers?

(ii) Sketch the area of the complex plane with numbers $z$ fulfilling $|z-2i| < 1$.

6. (i) Write down the definition of the exponential function in terms of an infinite series.

(ii) Express $\mathrm{Re}(e^{-ix})$ and $\mathrm{Im}(e^{-ix})$ by real functions.

(iii) Calculate $e^{(\pi/2)i}$, $e^{\pi i}$, $e^{2\pi i}$.

(iv) Calculate $f(t) \equiv \mathrm{Re}(e^{i\Omega t})$ where $\Omega = \omega + i\gamma$ and $t$, $\omega$ and $\gamma$ are real. What is the limiting value of $f(t)$ for $t \to \infty$ and positive $\gamma$? What is the limiting value of $f(t)$ for $t \to \infty$ and negative $\gamma$?

## Math Problems

7. Let $z_1 = 2+i$, $z_2 = 3-2i$.
(i) Draw $z_1$ and $z_2$ as vectors in the complex plane. Calculate $z = z_1 + z_2$ and draw $z$ as a vector in the complex plane.
(ii) Calculate $|z_1|$ and $|z_2|$ and explain their meaning.
(iii) Prove graphically that $|z_1 + z_2| \leq |z_1| + |z_2|$. Confirm this by direct calculation.
(iv) What is the polar form for the complex number $z = x + iy$, $x$ and $y$ real?
(v) Express $z = 1+i$ in polar form. Check the result by drawing $z$ as a vector in the complex plane.
(vi) Sketch the region of complex numbers in the complex plane ($x$–$y$ plane) with $1 < |z| < 2$.

8. Use de Moivre's Theorem:
(i) Take the cube of a complex number $z$ in polar form and prove two identities for trigonometric functions from that.
(ii) Do the same for the fourth power of $z$.

**Reading for next week: FM, Chap. 3**