P201 Workshop 7, Week 7

Please follow the instructions of your supervisor regarding timing of these problems.

Physics

1. A charge $-e$ is moving in the x-y-plane in the potential $\Phi(x,y) = \frac{c}{\sqrt{x^2+y^2}}$, where c is a constant.

   (i) Make a sketch of $\Phi(x,y)$!

   (ii) Calculate the change $d\Phi(x,y)$ (total differential) of the potential if the charge moves from (x,y) to $(x+dx, y+dy)$.

   Calculate the change in energy, $-e\frac{d}{dt}\Phi(x(t),y(t))$, if the charge moves (iii) along a circle $(r\cos\omega t, r\sin\omega t)$, (iv) along a straight, radial line $(a+vt, a+vt), a, v, t > 0$.

Math Practise

2. Calculate
   (i) $\frac{\partial}{\partial x}\cosh(x^2+y^2)$; (ii) $\frac{\partial}{\partial y}\sinh[\sin(xy)]$.

3. (i) Calculate the gradient $\nabla f(x,y)$ of $f(x,y) = -(x^2+y^2)$.

   (ii) Sketch the 'vector field' of the gradients in the x-y-plane: make a sketch by 'attaching' small arrows $\nabla f(x_0,y_0)$ at points (x_0,y_0).

   (iii) From the sketch in (iv) , sketch the equipotential lines of $f(x,y)$.

4. (i) Calculate the gradient $\nabla f(x,y)$ of $f(x,y) = -x^2+y^2$.

   (ii) Sketch the 'vector field' of the gradients in the x-y-plane: make a sketch by 'attaching' small arrows $\nabla f(x_0,y_0)$ at points (x_0,y_0).

   (iii) From the sketch in b), sketch the equipotential lines of $f(x,y)$.

Math Problems

5. (i) Find the stationary points (x,y) (if there are any), defined by

$$\nabla f(x,y) = (0,0) \to (x,y) \quad \text{is a stationary point}$$

   of the following functions:

   a) $f(x,y) = y$; b) $f(x,y) = x$; c) $f(x,y) = -(x^4+y^4)$;
   d) $f(x,y) = xy$; e) $f(x,y) = x^4 - y^4$.

   (ii) * Discuss the case d), $f(x,y) = xy$, in some more detail by sketching the 'vector field' $\nabla f(x,y)$ in the x-y-plane.

6. The lowest order expansion (up to the linear terms) of a function $f(x,y)$ around $(x = x_0, y = y_0)$ is, in analogy with a function of one variable, given by

$$f_1(x,y) = f(x_0,y_0) + \frac{\partial f}{\partial x}(x_0,y_0)(x-x_0) + \frac{\partial f}{\partial y}(x_0,y_0)(y-y_0).$$

   Show that $f_1(x,y)$ defines a two-dimensional *plane* and sketch (or demonstrate in a 'three-dimensional model') this plane for $f(x,y) = x^2+y^2$, $(x_0,y_0) = (0,1)$.

7. * Consider the differential equation $y'(x) = -\frac{x}{y(x)}$.

   (i) Make a sketch in the upper x-y-halfplane: to each point (x_0,y_0), attach a small piece of a straight line (arrow) with slope $-\frac{x_0}{y_0}$.

   (ii) Argue how this sketch can be used to graphically solve the differential equation.

   (iii) Solve the differential equation exactly, and show that b) is consistent with that solution.

8. Consider the differential equation $y''(x) + 4y(x) = 2e^{i3x}$.

   (i) Write down the general solution of the homogeneous equation, using exponentials.

   (ii) Write a particular solution $y_p(x)$ of the inhomogeneous equation as $y_p(x) = Ce^{i3x}$ and determine the constant C.

   (iii) * Determine the general solution of the differential equation, and solve it for $y(0) = 1, y'(0) = 0$.

Reading for next week: FM, Chap. 16