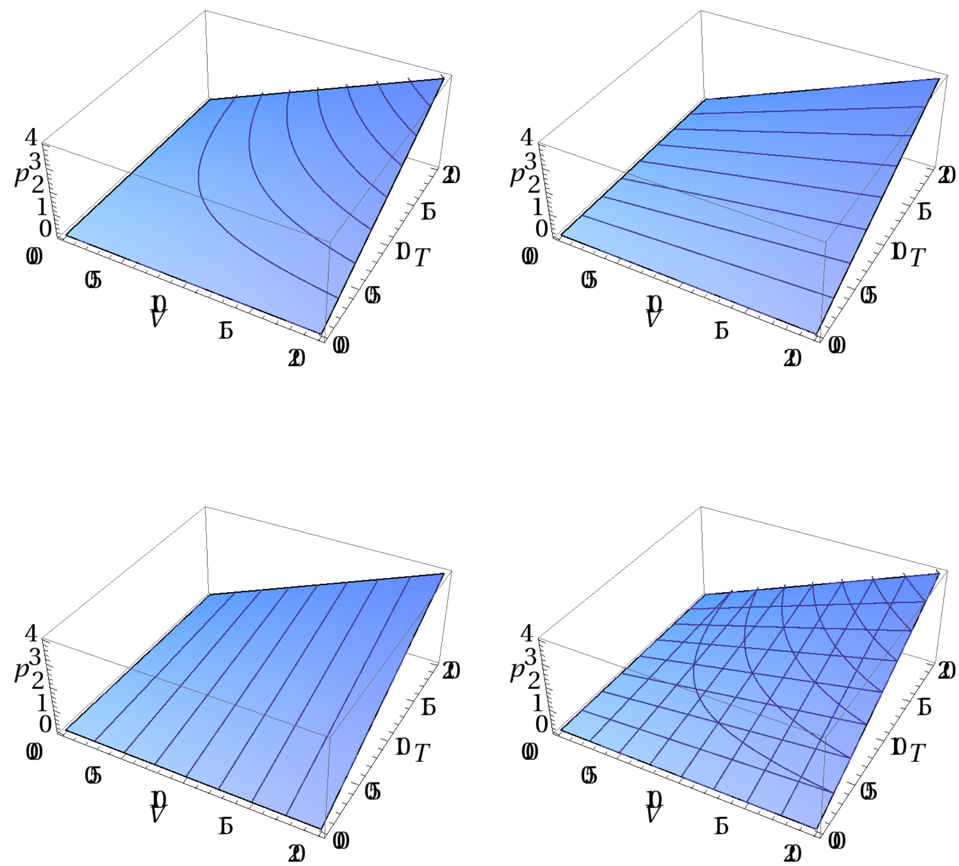
1. PVT surface

Plot3

```
plines = Table[ParametricPlot3D[{V, p/V, p}, {V, 0.001`, 2}, Disp
surf = Plot3D[V T, {V, 0, 2}, {T, 0, 2}, AxesLabel → {V, T, p}, Mes
Show[GraphicsGrid[{{Show[{surf, plines}, PlotRange → {{0, 2}, ·
```



2. Homogeneous DE

(i) Substitute $e^{z\,x}$, find $z^2 + 1 = 0$, i.e., $z = \pm i$. Conclude that $y(x) = C_1\, e^{i\,x} + C_2\, e^{-i\,x}$. Alternativ

(ii) Substitute $e^{z x}$, find $z^2 + z + 1 = 0$, i.e., $z_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1}{2} \pm i \frac{\sqrt{3}}{2}$. Conclude that $y(x) =$

$y(0) = 0 = C_1 + C_2 \Rightarrow C_1 = -C_2$.

$y'(0) = C_1 z_1 + C_2 z_2 = C_1(z_1 - z_2) = C_1 i \sqrt{3} = 1 \Rightarrow C_1 = -i \frac{\sqrt{3}}{3}$.

$y(x) = -i \frac{\sqrt{3}}{3} e^{-x/2}\left(\exp\left(i \frac{\sqrt{3}}{2} x\right) - \exp\left(-i \frac{\sqrt{3}}{2} x\right)\right) = -2 \frac{\sqrt{3}}{3} e^{-x/2} \sin\left(\frac{\sqrt{3}}{2} x\right)$.

(iii)Substitute $e^{z x}$, find $z^2 + 2 z + 1 = 0$, i.e., $z_{1,2} = -1$. Conclude that (special case) $y(x) = (C_1$

$y(0) = 0 = C_1 \Rightarrow C_1 = 0$.

$y'(0) = -C_1 + C_2 = C_1 = 1 \Rightarrow C_1 = 1$.

$y(x) = x e^{-x}$.

(iv) Substitute $e^{z x}$, find $z^2 + 3 z + 1 = 0$, i.e., $z_{1,2} = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3}{2} \pm \frac{\sqrt{5}}{2}$. Conclude that $y(x)$

$y(0) = 0 = C_1 + C_2 \Rightarrow C_1 = -C_2$.

$y'(0) = C_1 z_1 + C_2 z_2 = C_1(z_1 - z_2) = C_1 \sqrt{5} = 1 \Rightarrow C_1 = \frac{\sqrt{5}}{5}$.

$y(x) = \frac{\sqrt{5}}{3} e^{-3 x/2}\left(\exp\left(\frac{\sqrt{5}}{2} x\right) - \exp\left(-\frac{\sqrt{5}}{2} x\right)\right) = 2 \frac{\sqrt{3}}{3} e^{-x/2} \sinh\left(\frac{\sqrt{5}}{2} x\right)$.

3.

(i) Substitute $e^{z x}$, find $z^2 + 5 z + 4 = 0$, i.e., $z_{1,2} = \frac{-5 \pm \sqrt{25-16}}{2} = \frac{-5}{2} \pm \frac{3}{2} = -4, -1$. Conclude t

(ii)Substitute $C e^{z x}$, find $C\left(z^2 + 5 z + 4\right) e^{z x} = e^{2 x}$. Conclude $z = 2$, $C = \frac{1}{18}$.

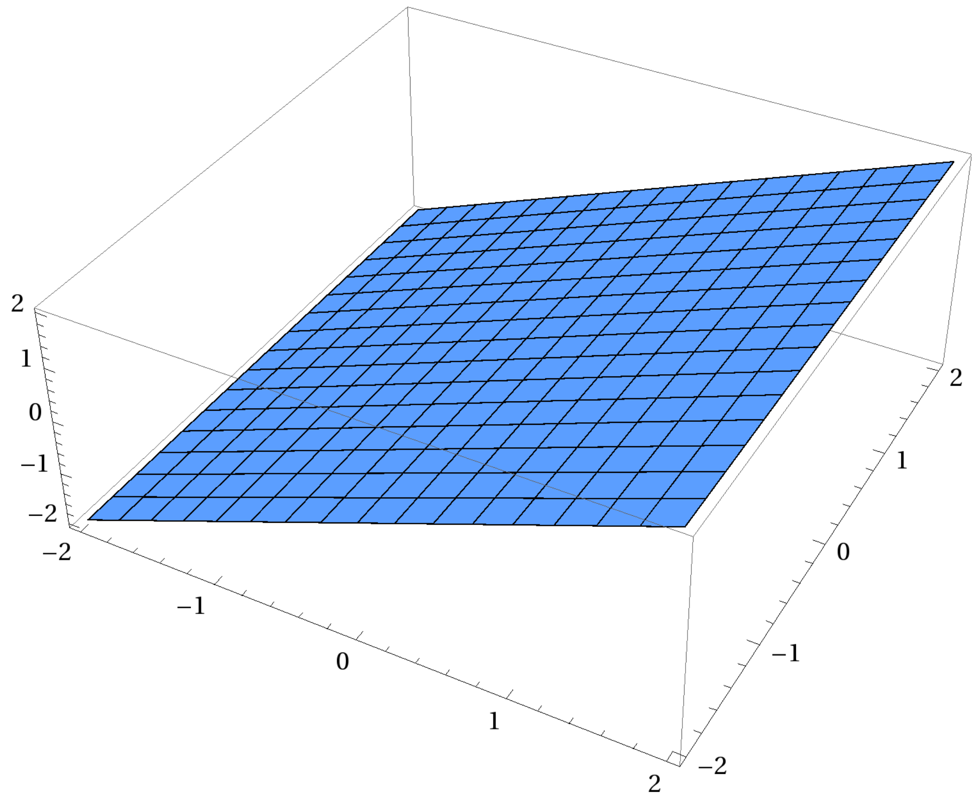
(iii) General solution is $y(x) = \frac{1}{18} e^{2 x} + C_1 e^{-x} + C_2 e^{-4 x}$. Use $y(0) = \frac{1}{18} + C_1 + C_2 = 0$ and y'

$y(x) = \frac{e^{-4 x}}{18} - \frac{e^{-x}}{9} + \frac{e^{2 x}}{18}$

4.

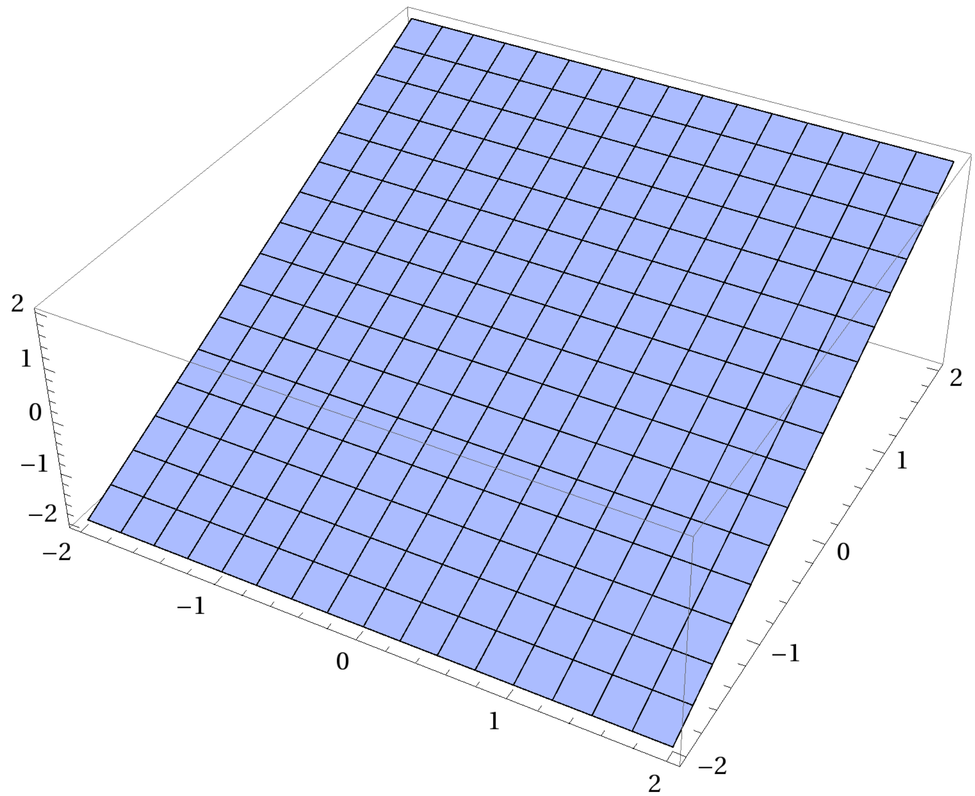
(i)

```
Plot3D[x, {x, -2, 2}, {y, -2, 2}]
```



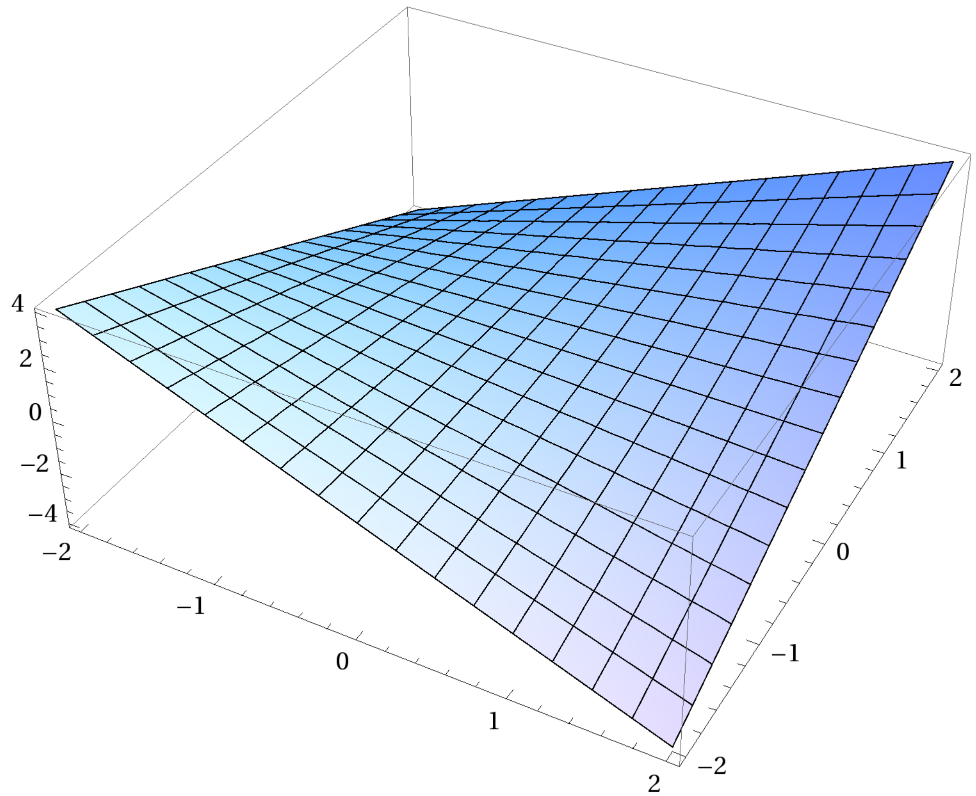
(ii)

```
Plot3D[y, {x, -2, 2}, {y, -2, 2}]
```



(iii)

```
Plot3D[x y, {x, -2, 2}, {y, -2, 2}]
```



5.

(i) $-2\,x\,e^{-\left(x^2+y^2\right)}$

(ii) $3\,x^2\,y^2\cos\left(x+x^2\,y^3\right)$

(iii) $\frac{\partial}{\partial x}3\,x\,y^2 = 3\,y^2$

(iv) $\frac{\partial}{\partial y}\left(2\,x+y^3\right) = 3\,y^2$

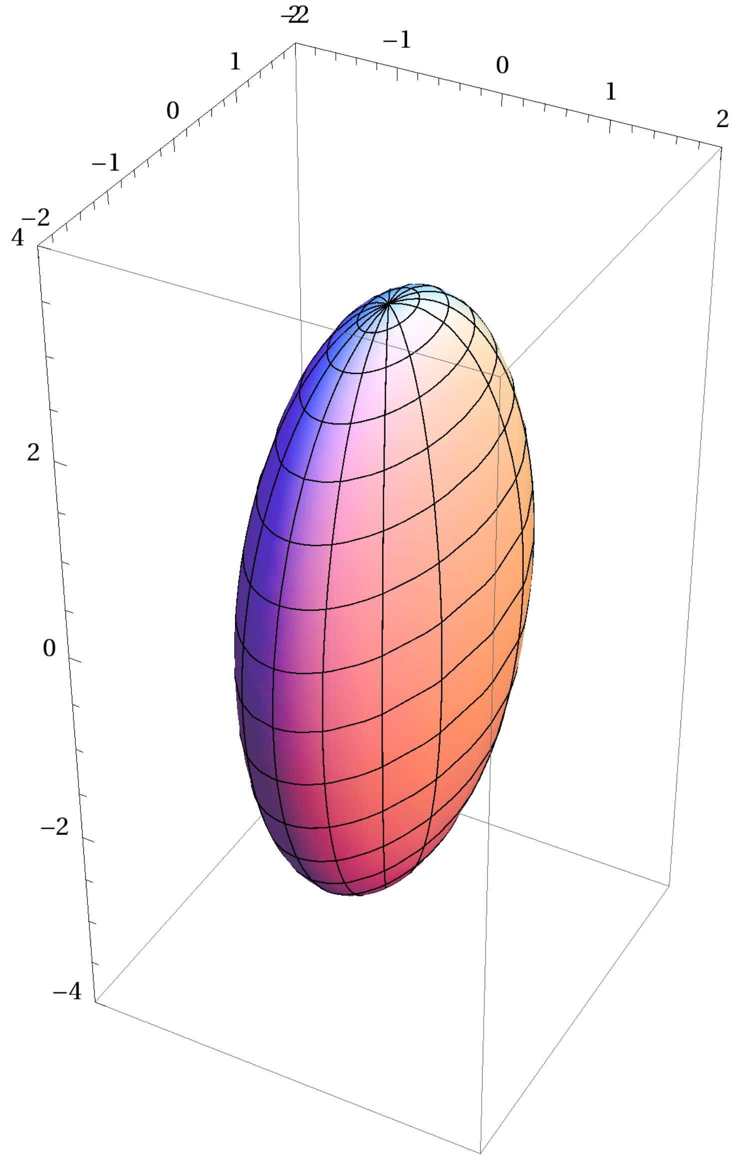
6.

(i) Use $a = 1$, $b = 2$, $c = 3$:

Simplify$\left[(1\cos(\phi)\sin(\psi))^2 / 1 + (2\sin(\phi)\sin(\psi))^2 / 2^2 + (3\cos(\psi))^2 / 3^2\right]$

```
1
```

```
ParametricPlot3D[{Cos[ϕ] Sin[ψ], 2 Sin[ϕ] Sin[ψ], 3 Cos[ψ]}, {ϕ,
```

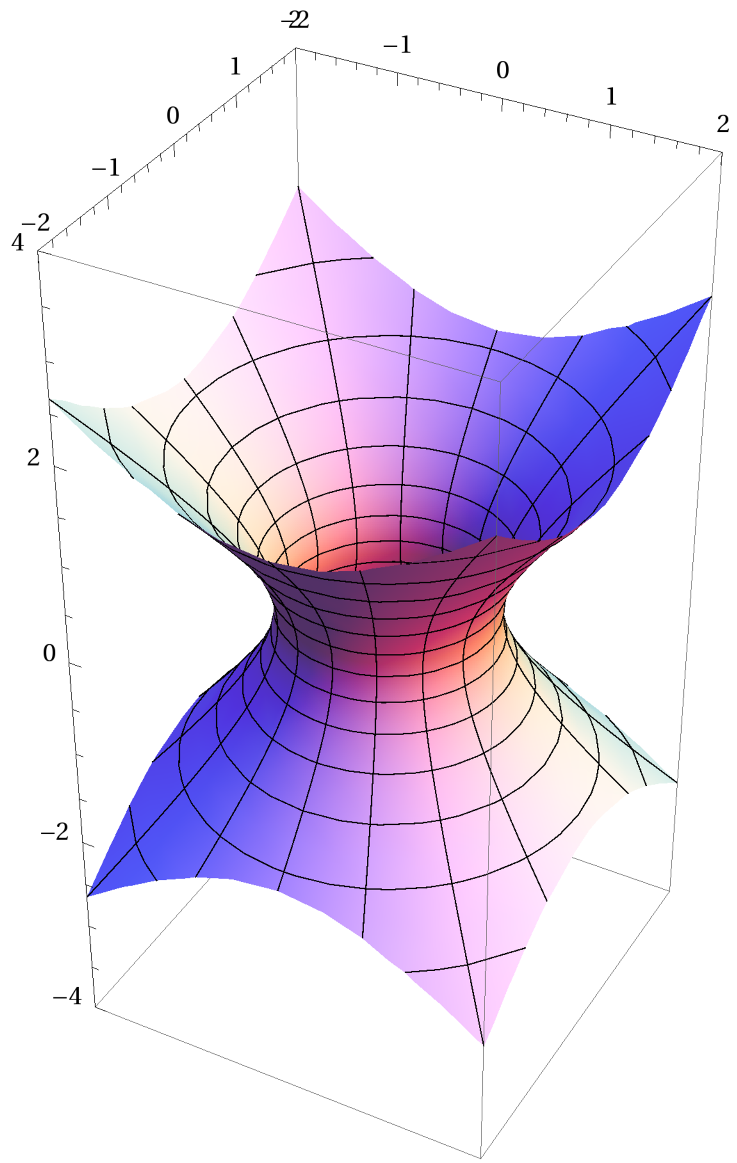


(ii) Use $a = 1$, $b = 1$, $c = 1$:

Simplify$\left[(a\cos(\phi)\cosh(\psi))^2 / a^2 + (b\sin(\phi)\cosh(\psi))^2 / b^2 - (c\sinh(\psi))^2 / c^2\right]$

```
1
```

```
ParametricPlot3D[{Cos[ϕ] Cosh[ψ], Sin[ϕ] Cosh[ψ], 1 Sinh[ψ]}, {
```

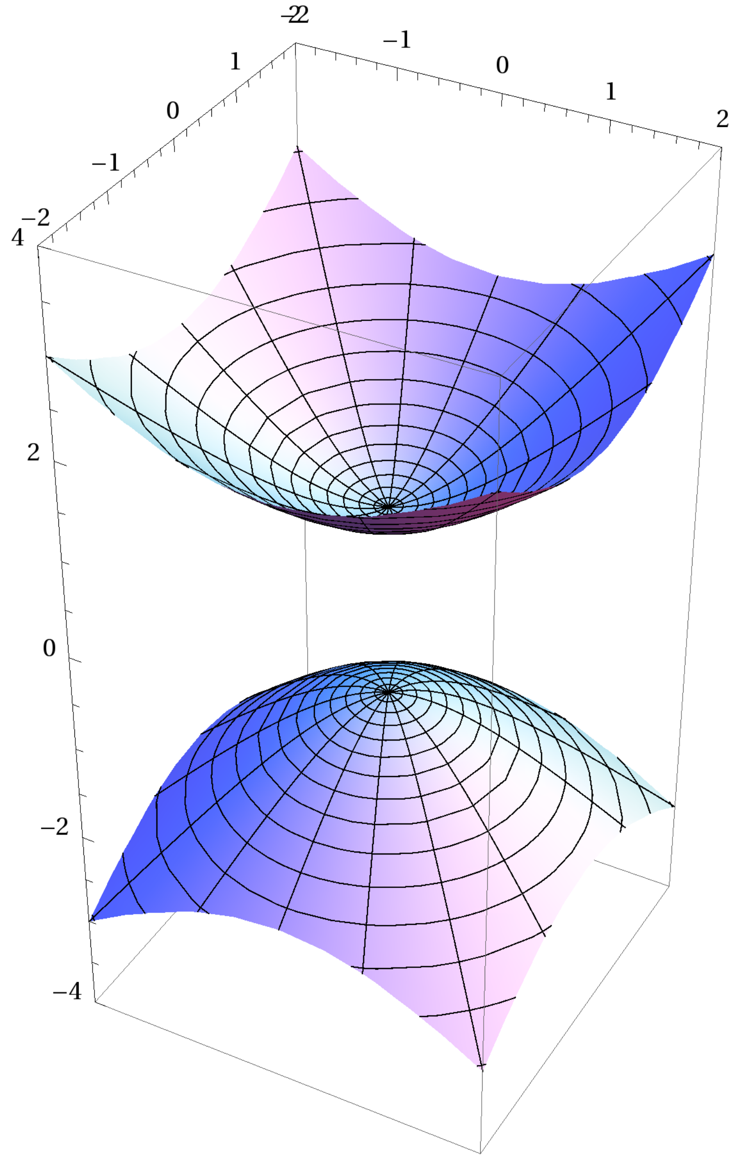


(ii) Use $a = 1$, $b = 1$, $c = 1$:

$$\textbf{Simplify}\left[(a\cos(\phi)\sinh(\psi))^2 \big/ a^2 + (b\sin(\phi)\sinh(\psi))^2 \big/ b^2 - (c\cosh(\psi))^2 \big/ c^2\right]$$

```
-1
```

```
ParametricPlot3D[{{Cos[ϕ] Sinh[ψ], Sin[ϕ] Sinh[ψ], 1 Cosh[ψ]},
```

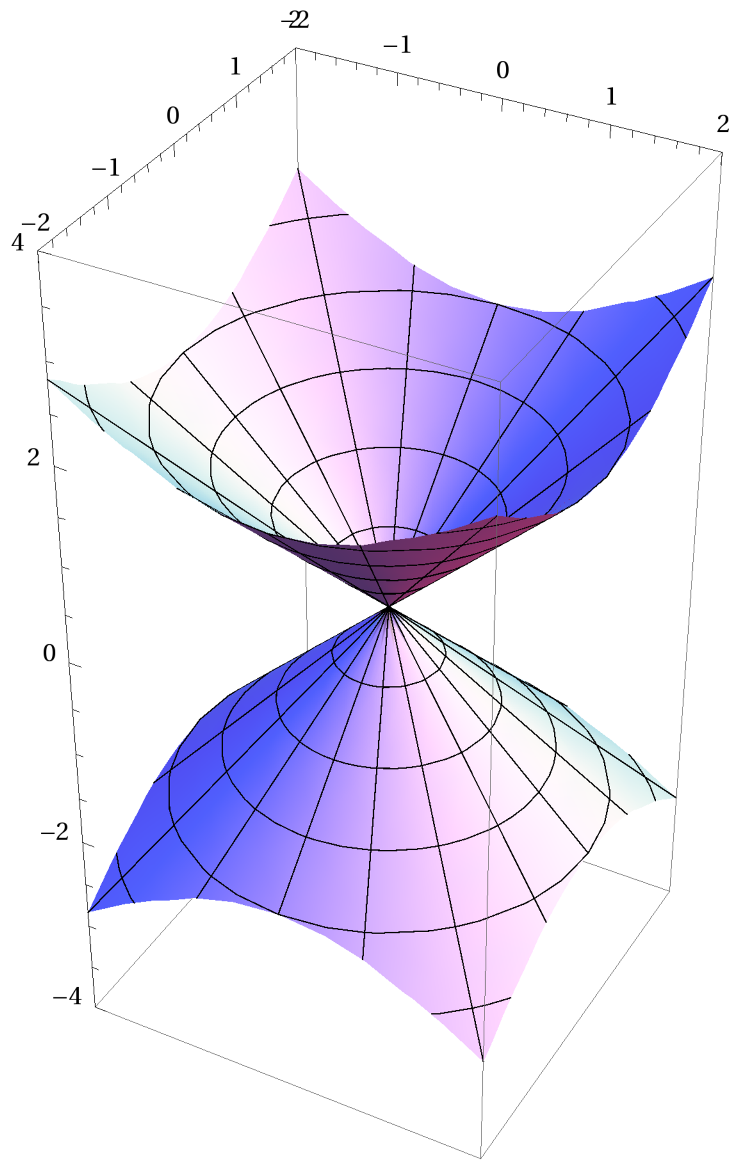


(ii) Use $a = 1$, $b = 1$, $c = 1$:

Simplify$\left[(a\cos(\phi)\,\psi)^2 / a^2 + (b\sin(\phi)\,\psi)^2 / b^2 - (c\,\psi)^2 / c^2\right]$

```
0
```

```
ParametricPlot3D[{Cos[ϕ] ψ, Sin[ϕ] ψ, 1 ψ}, {ϕ, 0, 2 π}, {ψ, -4, 4
```

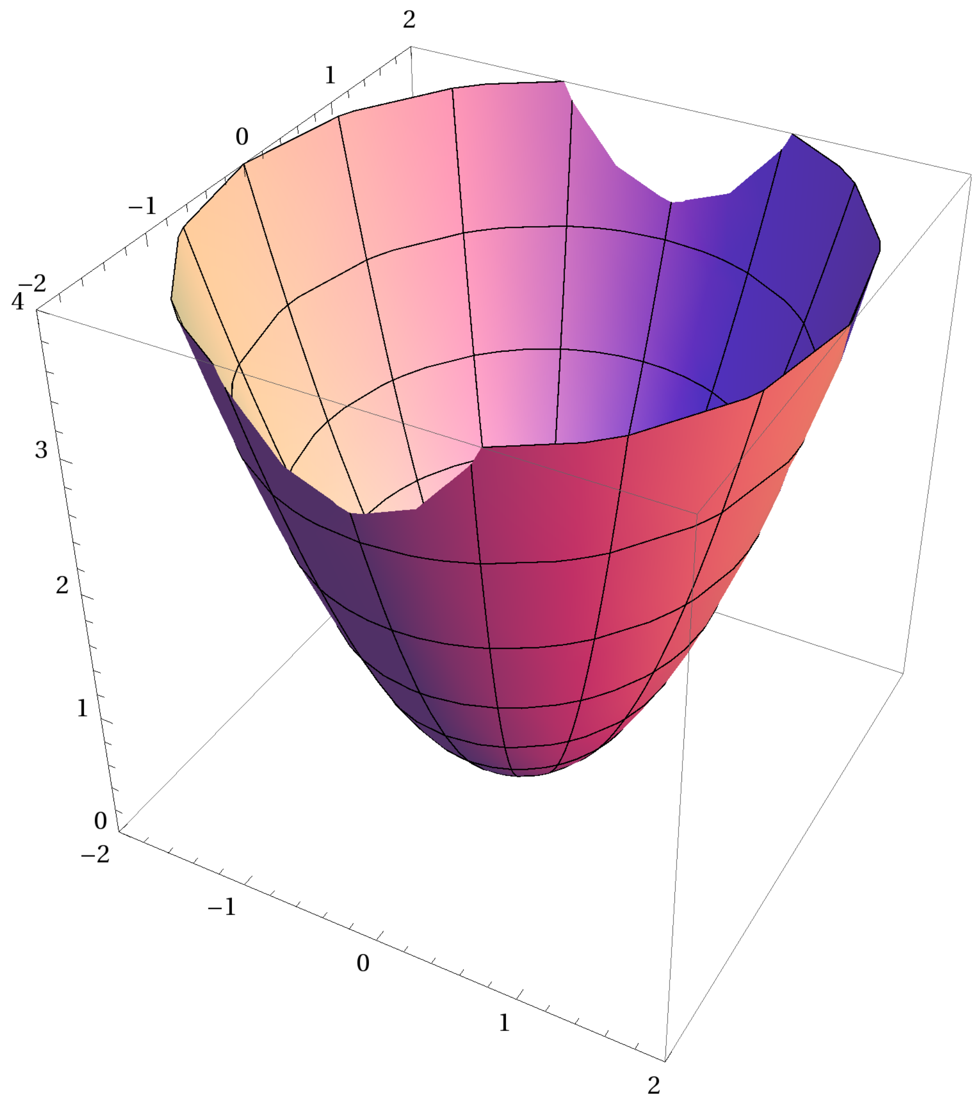


(ii) Use $a = 1$, $b = 1.1$:

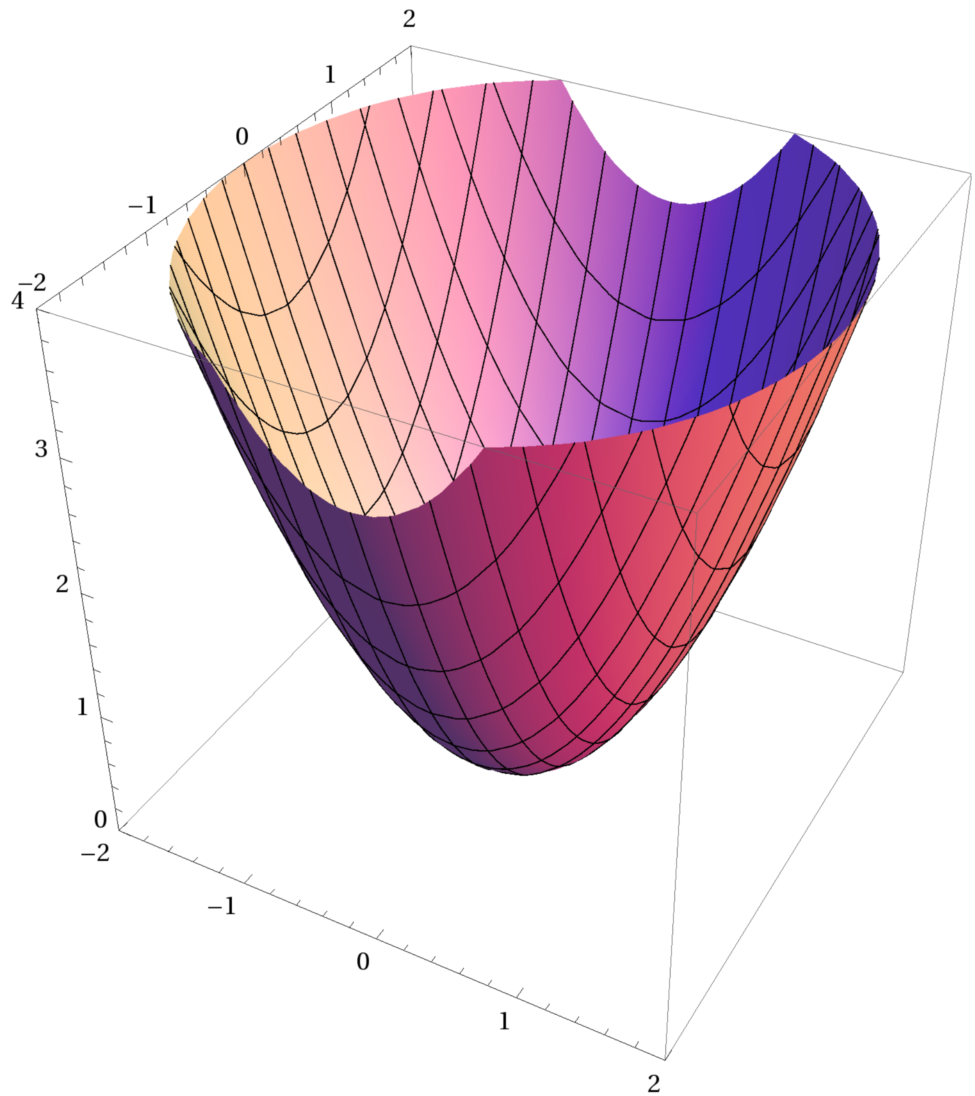
Simplify$\left[(a\cos(\phi)\,\psi)^2 \big/ a^2 + (b\sin(\phi)\,\psi)^2 \big/ b^2 - \psi^2\right]$

```
0
```

```
ParametricPlot3D[{Cos[ϕ] ψ, 1.1` Sin[ϕ] ψ, ψ^2}, {ϕ, 0, 2 π}, {ψ, (
```



```
ParametricPlot3D[{x, y, x^2 + y^2/1.1`^2}, {x, -2, 2}, {y, -2, 2}, Plot
```



(ii) Use $a = 1$, $b = 1.1$:

Simplify$\left[(a\cosh(\phi)\,\psi)^2 \big/ a^2 - (b\sinh(\phi)\,\psi)^2 \big/ b^2 - \psi^2\right]$

0

ParametricPlot3D$\left[\left\{x,\ y,\ x^2 - \frac{y^2}{1.1\grave{}^2}\right\},\ \{x,\ -2,\ 2\},\ \{y,\ -2,\ 2\},\ \text{Plot}\right.$