Workshop 1, Week 1

Please follow the instructions of your supervisor regarding timing of these problems.

1. (i) Find the solution of $\dfrac{dz}{dt} = 1 + z^2$ that satisfies $z = 1$ at $t = 0$.
(ii) Find the general solution of $y' = \sin^2(y)$.
the condition $y(0) = 1$.
(iii) Find the general solution of $(y')^2 = 1 - y^2$.

2. Solve $xy' - 2x^2 y = 2x^2 e^{2x^2}$.

3. (i) Find the general solution of $(x-1)(x-2)y' = xy$,
(ii) Find the solution of $yy' \cos^2(x) = 2 + \tan(x)$ that satisfies $y = 2$ at $x = \pi/4$.

4. The differential equation

$$\frac{d^2x}{dt^2} = ge^{-kt}$$

(g acceleration due to gravity, k a positive constant) describes the position of a falling parachutist. "Classify" this equation and provide the general solution. Now assume that the initial position is h and velocity is zero, and find the special solution.

5. Solve

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0 \quad .$$

6. Find the general solution to the DE ($\omega_0^2 \neq \Omega^2$)

$$\frac{d^2y}{dt^2} + \omega_0^2 y = A\cos(\Omega t) \quad .$$

7. The Schrödinger equation for a constant potential is given by

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0\psi = E\psi \quad .$$

Give the general solution to this equation. Which solutions are physically acceptable? Interpret the solutions.