Workshop 3, Week 3

Please follow the instructions of your supervisor regarding timing of these problems.

1. Write and classify the boundary and initial conditions governing the position $u(x,t)$ of a vibrating string of length L given that

	$x=0$	$x=L$	Initial displacement	Initial velocity
(i)	Fixed	Fixed	0	$g(x)$
(ii)	Free	Free	$\cos(x)$	0
(iii)	position=-slope	Fixed	$f(x)$	$g(x)$

2. A firework is set of in the middle of a spherical metal shell, generating a pressure wave, propagating with the speed of sound. If the metal shell remains intact, deduce the boundary condition(s) on the surface of the sphere, and give their classification.

3. A particle in a finite square well is described by the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2} + V(x)\phi = E\phi$$

The potential takes the form $V(x) = -V_0$ $|x| < a$, $V(x) = 0$ $|x| > a$.

(i) Give the form of the wave function for both $|x| < a$ and $|x| > a$, give that $-V_0 < E < 0$.

(ii) Treat $x = \pm a$ as a boundary. The physics requires that $\phi(x)$ and $\phi'(x)$ are continuous across the boundary, what conditions can you extract from these BCs?

(iii) Can you find a simple(r) way to write this type of BC?

4. Which of the following functions is periodic? If it is, give its period
a) x^2, b) $\tan(x)$, c) $\cos^2(x)$, d) 10, e) $|x|$, f) $x^2 - \text{int}(x^2)$.
"int" gives the smallest integer less than its argument.