Workshop 5, Week 5

Please follow the instructions of your supervisor regarding timing of these problems.

1. Find the Fourier sine and cosine series for the function defined on the half Fourier domain $(0, \pi)$ by

$$f(x) = 1, \text{ for } 0 < x < \pi/2 \qquad f(x) = 0, \text{ for } \pi/2 < x < \pi.$$

2. Use Parseval's theorem and the Fourier series for $f(\theta) = \theta^2$ on $(-\pi, \pi)$ to show that

$$1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \ldots = \frac{\pi^4}{90}.$$

3. An approximation to $f(\theta)$, $-\pi \le \theta < \pi$ is taken to be

$$F(\theta) = \frac{A_0}{2} + \sum_{n=1}^{N} \left(A_n \cos n\theta + B_n \sin n\theta\right).$$

Show that the mean square deviation

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \left[f(\theta) - F(\theta)\right]^2 d\theta$$

is

$$\frac{1}{4}\left(a_0 - A_0\right)^2 + \sum_{n=1}^{N} \frac{1}{2}\left[(a_n - A_n)^2 + (b_n - B_n)^2\right] + \sum_{n=N+1}^{\infty} \frac{1}{2}\left(a_n^2 + b_n^2\right).$$

(use Parseval's theorem). a_n and b_n are the Fourier coefficients of $f(\theta)$. Deduce that the approximation F with the smallest deviation has $A_n = a_n$ and $B_n = b_n$, $n = 0, \ldots, N$.