Workshop 9, Week 9

Please follow the instructions of your supervisor regarding timing of these problems.

1. By using Frobenius' method, find the general solution to the following differential equations (you do not have to solve the recurrence relations).

$$\text{(i)} \qquad (2x + x^3)\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6xy = 0,$$

$$\text{(ii)} \qquad (x - x^2)\frac{d^2y}{dx^2} + (1 - 5x)\frac{dy}{dx} - 4y = 0.$$

2. The pressure inside a sphere of radius C is given by the wave equation in three dimensions. In spherical coordinates we thus find

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\left[\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial p}{\partial \theta}\right) + \frac{\partial^2 p}{\partial \phi^2}\right] = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}.$$

Assume that the pressure is spherically symmetric, independent of θ and ϕ. Find the solution to this problem by separation of variables, $p(r, t) = R(r)T(t)$. (The boundary condition is of von Neumann type: the normal derivative of the pressure on the surface of the sphere is 0.)