**QM I: Homework 2**

Please hand in **before** Friday, 27 November, 12 noon in the departmental office, H floor

**1.** a) Calculate the expectation value of $x$ and $x^2$ in the normalised wave function [10 Marks]

$$\phi(x) = \sqrt{\tfrac{1}{2}}e^{-|x|/2} \tag{1}$$

**Hint:**

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x > 0 \end{cases} . \tag{2}$$

b) Sketch the function $\phi$, its derivative and the second derivative. How do they behave at zero? Calculate the derivative of this wave function. What conclusion can you draw about the behaviour of the potential in the Hamiltonian of which $\phi(x)$ is an eigenfunction? [10 Marks]

c) Argue that the potential $V(x)$ in the Schrödinger equation must be proportional to $\delta(x-0)$. [5 Marks]

**Bonus marks:** Find the proportionality constant. [5 Marks]

**2.** We study the reflection and transmission coefficients for a square barrier extending from $x=-a$ to $x=a$, with height $V_0$. The Schrödinger equation is

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\phi(x) + V(x)\phi(x) = E\phi(x), \tag{3}$$

with the potential

$$V(x) = \begin{cases} 0 & x < -a & \text{(region I)} \\ V_0 & -a < x < a & \text{(region II)} \\ 0 & x > a & \text{(region III)} \end{cases} . \tag{4}$$

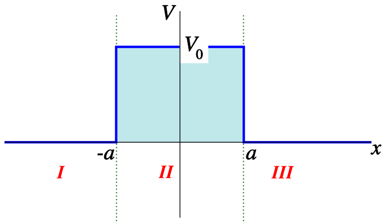


Figure 1: The square barrier.

Assume that the energy $E$ is *larger* than the height of the barrier.

a) What are the wave functions? [9 Marks]

b) What are the matching conditions? [5 Marks]

c) What is their explicit form? [5 Marks]

d) Sketch a strategy to find the solution for the transmission coefficient. [6 Marks]

**3.** Consider the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\phi(x) + V(x)\phi(x) = E\phi(x), \tag{5}$$

with the potential

$$V(x) = \begin{cases} 0 & x < -a & \text{(region I)} \\ -V_0 & -a < x < 0 & \text{(region II)} \\ \infty & x > 0 & \text{(region III)} \end{cases} . \tag{6}$$

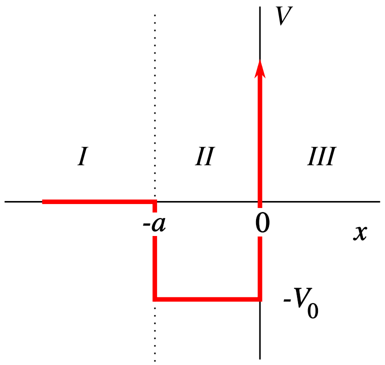


Figure 2: The potential.

a) Assume $-V_0 < E < 0$, where we look for bound (normalisable) states. What is the form of the wave function in regions I, II? Explain why the wave function in region III is zero. [5 Marks]

b) What are the (continuity) conditions satisfied by the wave function? Show that the wave function in region II can be written in the form $\phi(x) = A_2 \sin(\kappa x)$. Find an implicit relation defining the eigen energies. [8 Marks]

c) Assume $E > 0$. What are the values of the transmission and reflection coefficients for a plane wave coming in from the the left? [2 Marks]

d) For $E > 0$ we send in an incoming plane wave, $\phi(x) = \exp(ikx)$. Assume that the reflected wave can be written as $\phi_R(x) = A\exp(-ikx))$, and evaluate $A$ as a function of $V_0$ and $E$. [10 Marks]