

Workshop 12, Week 12

Please follow the instructions of your supervisor regarding timing of these problems.

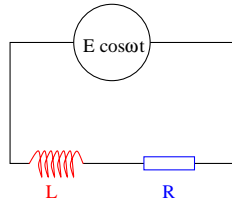
Math Review

1. * Given that $\sin'(x) = \cos(x)$, $\cos'(x) = -\sin(x)$, $\ln'(x) = 1/x$, differentiate:

$$\begin{array}{ll} \text{(i)} & \sin(x^2)/x^3, & \text{(ii)} & \tan(\cos(x)), \\ \text{(iii)} & \sin(\sqrt{x^2 + a^2}), & \text{(iv)} & x^2 \ln(x^2 + 4x). \end{array}$$

Physics Problems

2. * A simple electrical circuit consisting of an inductor (impedance L) and a resistor (resistance R) is driven by an AC voltage $E \cos(\omega t)$.



The current through the circuit satisfies

$$L \frac{dI}{dt} + RI = E \cos(\omega t).$$

Solve this differential equation.

3. A stone of mass m is thrown upwards with velocity $v_z(0) = u_z$. The stone is subject to the force of gravity, $-mg$, in the z direction, and a resistive force of magnitude $-mkv_z$. The stone's velocity $v_z(t)$ satisfies

$$\frac{dv_z}{dt} = -g - kv_z$$

Find the general solution to this equation:

- By using separation of variables (inversion);
- By using an integrating factor;
- Find the specific solution satisfying $v_z(t = 0) = u_z$.

Maths Practice

4. Solve the following for $y(x)$:
- $(\sec x)y' + y = \cos^2 x$; $y = 1$ at $x = 0$,
 - $y' + y/x = \sin x$; $y = 0$ at $x = \pi$,
 - $(\cos x)y' + (\sin x)y = \cos^2 x \operatorname{cosec} x$,
 - $y' + y \ln(x) = e^{-x \ln(x)}$.
5. Solve the following for $y(x)$:
- $y' = \frac{x+y}{x-y}$,
 - $y' + \frac{y(x+y)}{x(x-y)} = 1$; $y = 0$ at $x = 1$,
6. Solve the following for $y(x)$:
- $y' + 4y = 17 \sin x$,
 - $25yy' - 9x = 0$,
 - $4xyy' = y^2 - x^2$,

7. A differential equation of the form

$$\frac{dy}{dx} + y = f(x)y^n,$$

where n is a constant is called a Bernoulli equation.

- For what value(s) of n is the equation linear?
- For what value(s) of n is the equation separable?
- Show that the substitution $y = v^m$, where $m = 1/(1-n)$, transforms Bernoulli's equation to a linear one.

Have a good Xmass holiday & success with the exam.