P101   Workshop 2, Week 2

Please follow the instructions of your supervisor regarding timing of these problems.

Physics Problems

1. * The dangerous[1] radioactive isotope Strontium-90 was deposited all over Europe after the Chenobyl accident. Given that radioactive decays are exponential, and the half-life is 27 years,
   (i) Show that the number of radioactive isotopes at time t can be expressed in the formula $N(t) = N_0 e^{-t/t_0}$.
   (ii) Express t_0 in terms of the half life t_h.
   (iii) Find the time in years after which only 1/1000th of the original radioactive material remains.

2. * According to Kepler's law of planetary motion, the orbital period T of a planet is related to its average distance R from the sun by $T^2 = kR^3$.
   (i) Explain why plotting $\log_{10} T$ against $\log_{10} R$ is useful to test this relationship.
   (ii) What is the gradient of the graph in the previous problem (i)?
   (iii) How does the result differ if we plot $\log_e T$ against $\log_e R$?

3. * The intensity level of sound is usual give in dB (decibels), and is defined as the logarithm of the intensity of the sound by $\beta = (10\,\text{dB})\log_{10}(I/I_0)$, where I_0 is the threshold of hearing, $I_0 = 10^{-12}\,\text{W/m}^2$.
   (i) When a person wears a hearing aid, the intensity level is increased by 30 dB. By what factor does the sound intensity increase?
   (ii) The pain threshold is 130 dB. Calculate the intensity, and the power on the eardrum, assuming a surface area of 10 mm^2.

4. When a single person shouts at a football match, the sound intensity level at the middle of the field is 60 dB. When all people shout together, the intensity level rises to 109 dB. Assuming that each person generates the same sound intensity at the centre of the field, how may people are at the game?

5. A sound source emits sound uniformly in all directions. A straight line is drawn from the source. On this line, determine the position of two points, 1 m apart, such that the intensity level at one point is 2.0 dB greater than at the other.

[1]It can replace calcium in bone tissue.

Maths Review

6. Sketch graphs of

   * (i) $\log_2(x)$,
   (ii) $2(1 - e^{-x})$,
   * (iii) $\dfrac{(x-1)(x+6)}{(x+3)(x-4)}$,
   (iv) $\dfrac{1}{x^2 + 2x + 1}$,
   * (v) $\cos(x - \pi/4)$,
   (vi) $\operatorname{cosec}(x)$,
   * (vii) $\tan(2x)$,
   (viii) $3\sin(x + \pi/3)$.

Maths Practice

7. Evaluate, without using a calculator,

   (i) $\sin(0)$,
   (ii) $\cos(\pi)$,
   (iii) $\tan(2\pi)$,
   (iv) $\sin(-\pi/2)$,
   (v) $\sin(\pi/4)$,
   (vi) $\tan(\pi/2)$,
   (vii) $\cos(0)$,
   (viii) $\cos(-\pi/3)$,
   (ix) $\operatorname{cosec}(\pi/4)$,
   (x) $\sec(\pi/2)$,
   (xi) $\cot(-\pi)$.

8. Express the following in either $\sin(x)$ or $\cos(x)$

   (i) $\dfrac{\cos(2x) + \cos(x) + 1}{\cos(x)}$,
   (ii) $\sin(3x)$,
   (iii) $1/(1 + \tan^2(x))$,
   (iv) $\sin(2x + \pi/2)$.

Reading for next week: Chapter 4