Workshop 3, Week 3

Please follow the instructions of your supervisor regarding timing of these problems.

Maths Review

1. Solve the following for x

(i) $*$ $\dfrac{1}{x+3} = 5,$ (ii) $*$ $\dfrac{1}{\sqrt{1-x^2}+1} = 1/3,$

(iii) $*$ $x^2 + 5x - 14 = 0,$ (iv) $*$ $3x^2 + x - 2 = 0.$

2. Solve the following for x and y

(i) $\begin{cases} 2x + y = 6 \\ 3x - 5y = 2 \end{cases},$ (ii) $*$ $\begin{cases} 2x + y = k \\ 2x - 2y = 1 \end{cases}.$

3. Under which conditions does the equation $x^2 - rx + s = 0$ have
(i) two real roots, and
(ii) a repeated root.

Physics Problems

4. $*$ In SHM the motion of a spring is described by $x = 4\cos(4t)$ cm. Find the times where the spring is 2 cm from its equilibrium position.

5. $*$ The top of a tower, which is know to be a distance of 3km from here, appears under an angle of 1.1° with the horizon. Calculate the height of the tower.

6. $*$ A feather is moving under the influence of two loudspeakers, that both emit a pure tone of frequency ω. With only the first speaker turned on, the feather moves backwards and forwards as $A\cos(\omega t + \phi_1)$, with only the second as $A\cos(\omega t + \phi_2)$. Find the resulting motion of the feather if both are turned on.

7. When a violin string is plucked, it performs a standing wave of frequency 500 Hz. The speed of sound in the string is 300 m/s.
(i) Find the deviation of the string from equilibrium as a function of time t and position along the string (assuming the maximal initial change from equilibrium is 2mm).
(ii) Estimate the length of the violin.
(iii) Show, using an appropriate formula, how this can be written as a sum of left and right moving waves.

8. Aliens invade the earth, and stop it orbiting around the sun. It now makes a freefall to the sun. As discussed on http://faculty.trumbull.kent.edu/gallant/freefall.htm, the time we have left is

$$\tau = \sqrt{\rho(\rho-1)} + \tfrac{1}{2}\rho^{3/2}\left(\frac{\pi}{2} + \arcsin\frac{\rho-2}{\rho}\right)$$

with $\tau = t\sqrt{2GM/R^3}$, $\rho = r/R$, where M, R are the radius and mass of the sun, and r the distance between the earth and the sun. Using $G = 6.670 \times 10^{-11}\mathrm{Nm^2kg^{-2}}$, $M = 1.989 \times 10^{30}$ kg, $R = 695{,}000$ km, $r = 150{,}000{,}000$ km, calculate the time it takes to reach the sun.

Maths Practice

9. Evaluate (without using a calculator)

$*$ (i) $\tan^{-1}(1),$ (ii) $\tan^{-1}(\sqrt{3}),$

$*$ (iii) $\tan^{-1}(1/\sqrt{3}),$ (iv) $\cos^{-1}(1/2),$

$*$ (v) $\cos^{-1}(1/\sqrt{2}),$ $*$ (vi) $\cos(\sin^{-1} x),$

(vii) $\tan(\sin^{-1} x),$ (viii) $\tan(\cos^{-1} x).$

10. Sketch the polar curves

$*$ (i) $r = \theta,$ (ii) $r = \cos(\theta),$

$*$ (iii) $r = 2\cos(3\theta),$ (iv) $r = 3/(2 + \cos(\theta)).$

No reading, but 1st is coursework due next Tuesday