

Workshop 5, Week 5

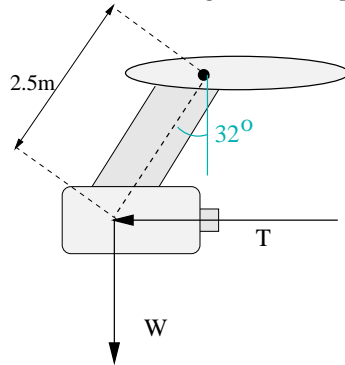
Please follow the instructions of your supervisor regarding timing of these problems.

Maths Review

- * (i) Determine x so that $\sin(x) < 1/2$.
(ii) Determine x so that $\sin^{-1}(x) > \pi/3$.
- Express the following in either $\sin(x)$ or $\cos(x)$
 - $\frac{\sin(2x) + \sin(x)}{\sin(x)}$, (ii) $\cos(3x)$, (iii) $1/(1 + \cot^2(x))$.

Physics Problems

- * A jet engine is suspended beneath the wing of an aeroplane, see figure.



The weight W of the engine is 102,000 N and acts downwards. In flight the thrust T of the engine is 62,300 N. Find the magnitude and the direction of the torque due (a) the weight and (b) the thrust, relative to the centre of the wing (black dot). Explain why the engine is not directly under the wing.

Reminder: The torque $\mathbf{T} = \mathbf{r} \times \mathbf{F}$ describes the “rotational effect” of a force.

- * A tennis ball ($m = 100$ g) is attached to a (massless) string.
 - Calculate the angular momentum if the ball traverses a 1m circle in 5 seconds.
 - The string is suddenly shortened to 10 cm; find the new velocity, and the time taken to traverse its circular orbit once.
- * An electron beam moves through a region of space where there is a uniform magnetic field, of magnitude 2.0 T, directed toward the positive z axis.

The electrons move with a velocity of magnitude 3×10^5 ms^{-1} , in the xz plane at an angle of 30° to the positive z axis. Find the force on a single electron ($q = e = -1.6 \times 10^{-19}$ C).

Hint: The magnetic force is $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.

- A straight horizontal wire carries a current of 50 A from west to east in a region between the poles of a large electromagnet, which provides a uniform magnetic field toward the northeast (i.e., 45° north of east) with magnitude 1.2 T. Find the direction of the force on a 1 m section of wire.
Hint: The magnetic force on a current carrying wire is $\mathbf{F} = I\mathbf{l} \times \mathbf{B}$.
- A small Styrofoam pellet of mass 1 mg and with charge 10 nC moves in a uniform magnetic field of strength 1 T, parallel to the $+z$ -axis. If the initial velocity of the pellet is 0.01 m/s parallel to the x axis, and 0.1 m/s in the $+z$ direction, find
 - The size and direction of the force on the particle.
 - The orbit of the particle.
 - Answer the same questions if an additional electric field of 10 V/m is applied in the $+z$ direction.**Hint:** The magnetic force is $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.

Math Practise

- Given $\mathbf{a} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = \mathbf{j} + 3\mathbf{k}$, $\mathbf{c} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, determine

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|---|---|
| (i) * $\mathbf{a} \times \mathbf{b}$ | (ii) $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$ |
| (iii) * $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a}$ | (iv) $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{b} \times \mathbf{c})$ |
| (v) * $(\mathbf{a} \cdot \mathbf{b})(\mathbf{a} \times \mathbf{c})$ | |

- Find a vector of magnitude 5 perpendicular to $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and to $\mathbf{b} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Prove that this vector is perpendicular to $\mathbf{c} = \lambda\mathbf{a} + \mu\mathbf{b}$ where λ and μ are any scalars.

Reading for next week: Chapter 6 (Exps and Logs)