

Workshop 6, Week 6

Please follow the instructions of your supervisor regarding timing of these problems.

Math Practice

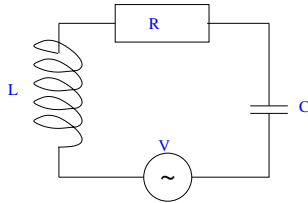
1. Solve the following for x

(i) * $-2 \exp(x) + \exp(2x) = -1$, (ii) * $\ln(1/(1+x^{1/2})) - \ln(y^{1/3}) = 2$.

Physics Problems

2. Under the influence of wind, a cannonball moves along the orbit $x = 18t - 3t^2$ m, $y = 20t - 5t^2$ m. Find dy/dx and dx/dy for this orbit, and by calculating values at suitable points, make a sketch of the orbit.

3. In a simple circuit (consisting of a coil, a resistor and a capacitor in series), see the figure,



L , R , and C are positive constants and the magnitude of the impedance Z (the ratio of the voltage and current amplitudes) is given in terms of the frequency ω of the applied external AC field as

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \text{ Ohm.}$$

As ω is varied, find the position and value of the maxima and minima of Z .

4. A planet of mass m moves under the influence of the gravitational force of the sun (mass M), $\mathbf{F} = -\frac{GmM}{r^2}\mathbf{r}$. Show that the angular momentum, $\mathbf{L} = m\mathbf{r} \times \mathbf{v}$, is constant in time.

Hint: Note that $\mathbf{r} = \mathbf{r}(t)$, and express the acceleration in terms of \mathbf{r} .

5. Evaluate the derivative of the following functions without use of a formula sheet

(i) * $\sin^{-1}(x)$, (ii) $\cos^{-1}(x)$, (iii) $\tan^{-1}(x)$.

6. Differentiate the following functions with respect to x of

(i) * $\sin^{-1}(2x)$, (ii) $\tan^{-1}(x/5)$, (iii) $\ln(ax^2 + bx + c)$.

7. * Using the product, quotient or chain rule, differentiate the following with respect to x :

(i) $\frac{\sin(x)}{x}$, (ii) $\frac{(x^2 + 1)(x + 2)}{(x^2 + 2)(x + 1)}$, (iii) $3^{\ln x}$.

8. Find the first two derivatives (with respect to x) of

(i) * $\sin(5x^2)$, (ii) $\frac{x}{x+1}$, (iii) $(x^2 + 4)^{1/2}$.

9. * Given that $y = x^2 e^x$ find

(i) $\frac{dy}{dx}$, (ii) $\frac{d^2y}{dx^2}$, (iii) $\frac{d^3y}{dx^3}$.

Maths Practice

No reading for next week: Coursework!