

Workshop 8, Week 8

Please follow the instructions of your supervisor regarding timing of these problems.

Math Practice

1. * The vectors \mathbf{a} and \mathbf{b} point south and west, respectively. The length $|\mathbf{a}| = 2$ cm and $|\mathbf{b}| = 1$ cm. Draw rough sketches to show the vectors

$$(i) -\frac{1}{2}\mathbf{a}, \quad (ii) \mathbf{a} - \mathbf{b}, \quad (iii) 2\mathbf{a} + 2\mathbf{b}.$$

Physics Problems

2. * (i) The current in a circuit is known to be 10.00 A (error negligible). We measure the voltage over a resistor to be 20.0 ± 0.1 V. Determine the resistance and its error.

Hint: Use $V = IR$.

(ii) Estimate the error in the resistance when the potential is known accurately, as $V = 20.00$ V, and the current is measured as $I = 1.00 \pm 0.05$ A.

Hint: Use the estimate for the derivative $\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$ if δx is small.

3. * The Coulomb force between two electrons is given by $\mathbf{F} = \frac{ke^2}{r^3}\mathbf{r}$
- (i) Calculate the work done by moving one charge over a small distance, from \mathbf{r} to $\mathbf{r} + \delta\mathbf{r}$, parallel to \mathbf{r} .
- (ii) Use the result above to find an expression for the work done by moving one charge from ∞ to \mathbf{R} . Could you have predicted the result?
- (iii) Find the work done by moving the charge from \mathbf{R} to zero distance.

4. The electric voltage from your wall outlet satisfies $V(t) = V_0 \cos(\omega t)$, with $\omega = 2\pi f$, and the frequency $f = 50$ Hz. The root-mean-square (rms) voltage is defined as the average over one period,

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T V(t)^2 dt}.$$

- (i) Find T ;
(ii) Calculate the relation between V_0 and V_{rms} ;
(iii) Give that $V_{\text{rms}} = 240$ V, determine V_0 .

Maths Practice

5. * Find the minimum and maximum of

$$f(t) = \frac{1}{\sqrt{1 + \sin^2 \omega t}}$$

for t from 0 to $2/\omega$, and sketch this function.

6. * Evaluate the following

$$(i) \int \frac{1}{x^2} dx, \quad (ii) \int (5x^3 + 3x^4) dx,$$
$$(iii) \int \frac{1}{\sqrt{1-x^2}} dx, \quad (iv) \int \frac{1}{1+x^2} dx,$$
$$(v) \int \frac{1}{x} dx, \quad (vi) \int \frac{1}{\sqrt{x^5}} dx.$$

7. * Evaluate

$$(i) \int_1^2 x dx, \quad (ii) \int_1^3 x^3 dx,$$
$$(iii) \int_0^5 \sqrt{x} dx, \quad (iv) \int_{-1}^1 \frac{1}{x^4} dx,$$
$$(v) \int_0^\infty e^{-x} dx, \quad (vi) \int_{-\infty}^\infty \frac{1}{1+x^2} dx.$$