

Workshop 9, Week 9

Please follow the instructions of your supervisor regarding timing of these problems.

Math Review

1. * A particle is moving with velocity $\mathbf{v} = (1, 3, 4)$ m/s and at time $t = 0$ is at the point P with position vector $\overrightarrow{OP} = (2, 3, -2)$ m. What is the coordinate vector $\mathbf{r}(t)$?

Physics Problems

2. * The Biot-Savart law states that the magnetic field by a current carrying wire is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int dl \frac{\mathbf{I}(l) \times \mathbf{r}}{r^3}.$$

Use this relation to find the magnitude and direction of the magnetic field on the axis of symmetry (axis through the midpoint) of a circular loop of wire.

3. A parabolic Kepler orbit is described by the polar curve

$$r = 1 / (1 + \frac{1}{2} \sin \theta).$$

Calculate the work done by the gravitational force, $\mathbf{F} = \frac{GMm}{r^3} \mathbf{r}$, over a full revolution.

4. Find the surface area of an ellipse of major axis a and minor axis b using integration.

Maths Practice

5. Evaluate the following integrals using substitution where necessary:

(i) * $\int x(2x^2 - 3)^5 dx$, (ii) $\int z\sqrt{(9 - z^2)} dz$,

(iii) * $\int \frac{1}{(3 - 2v)^2} dv$, (iv) $\int \frac{6}{\sqrt{4 - 5t}} dt$,

(v) * $\int_1^3 e^{-2x} dx$, (vi) $\int xe^{x^2} dx$,

(vii) * $\int \tan(3x) \sec(3x) dx$, (viii) $\int_{\pi/6}^{\pi/2} \frac{\cos^3 x}{\sin x} dx$.

6. Substituting a trigonometric function for x , evaluate:

(i) * $\int \frac{x^2}{\sqrt{4 - x^2}} dx$, (ii) $\int \frac{\sqrt{4 - x^2}}{x^2} dx$

7. Using integration by parts evaluate:

(i) * $\int 2xe^{-x} dx$, (ii) $\int x^2 \cos(4x) dx$,

(iii) * $\int \tan^{-1}(x) dx$, (iv) $\int (\ln x)^2 dx$,

(v) * $\int (x + 1)^{10}(x + 2) dx$, (vi) $\int_0^{\pi} e^{-x} \sin(2x) dx$.

No assigned reading for next week: Coursework!