Workshop 1, Week 1

Please follow the instructions of your supervisor regarding timing of these problems.

Curves (CALCULATORS NOT ALLOWED!)

1. (i) Sketch the curves (**Lorentzian**)

$$f_a(x) = \frac{1}{x^2+a^2}, \quad f_b(x) = \frac{1}{x^2+b^2}, \quad f_c(x) = \frac{1}{x^2+c^2}$$

for $a > b > c > 0$ within one coordinate system.

(ii) Calculate the constant N in $f(x) = N/[x^2+a^2]$ such that $\int_{-\infty}^{\infty} dx f(x) = 1$.

2. Sketch the curves (**Gaussian**)

$$g_a(x) = e^{-x^2/a^2}, \quad g_b(x) = e^{-x^2/b^2}, \quad g_c(x) = e^{-x^2/c^2}$$

for $a > b > c > 0$ within one coordinate system.

3. Sketch the curves (one graph each)

$$s(x) = \sin\left(\frac{1}{x}\right), \quad t(x) = \ln\left(\frac{1}{x}\right), \quad u(x) = \exp\left(-\frac{1}{x^2}\right).$$

4. Sketch the functions $a_i(t), i = 1,2,3$ (all into one graph)

$$a_1(t) = |t|, \quad a_2(t) = |t-1|, \quad a_3(t) = |t+1|.$$

5. * (i) Sketch the Planck distribution for the spectral energy density of **black body radiation**[1] as a function of the frequency ν,

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}.$$

Sketch different curves (each with one fixed temperature T).

(ii) Show that the total energy density $U(T) = \int_0^\infty d\nu\, u(\nu, T)$ is proportional to T^4 (Stefan–Boltzmann–law).

6. **Parametric curves**
A particle moves in the x–y–plane on a curve with coordinate $x(t)$ and $y(t)$, where the parameter t denotes the time. t shall run from $t = 0$ up to $t = \infty$. Sketch the following curves, including direction arrows:

$$\begin{aligned}
\text{(i) } x(t) &= 1, \quad y(t) = t \\
\text{(ii) } x(t) &= t, \quad y(t) = 2t \\
\text{(iii) } x(t) &= a\sin(\omega t), \quad y(t) = b\cos(\omega t), \quad \omega > 0, a > 0, b > 0 \\
\text{(iv) } x(t) &= \exp(-\gamma t)\sin(\omega t), \quad y(t) = \exp(-\gamma t)\cos(\omega t), \quad \omega > 0, \gamma > 0 \\
\text{(v) } x(t) &= \exp(-\gamma t)\sin(\omega t), \quad y(t) = \exp(-\gamma t)\cos(\omega t), \quad \omega > 0, \gamma < 0.
\end{aligned}$$

Math Practise

7. Calculate the following:
(i) $(-1+2i)(4-i)$;
(ii) $(2-i)(-7+22i)$;
(iii) $\dfrac{3-2i}{-1+i}$;
(iv) i^4;
(v) i^{30};
(vi) Find real numbers x and y such that $3x + 2iy - ix + 5y = 7 + 5i$.

8. (i) Find the complex solutions of $z^2 - 10z + 40 = 0$.
(ii) Define the function $f(x) = [x+i]^{-1}$ for real x. Calculate and sketch $f_r(x) := \mathrm{Re}[f(x)]$ and $f_i(x) := \mathrm{Im}[f(x)]$.
(iii) Show that for any complex number Ψ, the product $\Psi^*\Psi$ (where Ψ^* is the complex conjugate of Ψ) is real.

Other problems

9. Calculate
(i) $\int dx \ln(x)$;
(ii) $\int dx x e^{ax}$;
(iii) The volume of the solid of revolution if the curve $y(x) = 1 - x^2$, $-1 < x < 1$ is rotated around the y–axis;
(iv) $f'(x)$ where $f(x) = \int_0^x dy g^2(y)$ for a general $g(y)$.

Reading for next week: FM, Chaps. 1-2

[1] The radiation emitted by a totally absorbant body at temperature T