P201 Workshop 6, Week 6

Please follow the instructions of your supervisor regarding timing of these problems.

Physics

1. Consider the ideal gas equation

$$pV = nRT,$$

where n is the density of a gas of atoms in a volume V at temperature T, and R is a constant. Sketch the surface

$$p = p(V, T)$$

for constant density n. Skecth the following on the surface $p(V, T)$:
(i) curves of constant pressure p (isobars), (ii) curves of constant temperature T (isotherms), and
(iii) curves of constant volume V (isovolumes).

Math Practise

2. Solve the differential equations, and in each case show that the solution is real:
(i) $y''(x) + y(x) = 0, y'(0) = 1, y(0) = 0$
(ii) $y''(x) + y'(x) + y(x) = 0, y'(0) = 1, y(0) = 0$
(iii) $y''(x) + 2y'(x) + y(x) = 0, y'(0) = 1, y(0) = 0$
(iv) $y''(x) + 3y'(x) + y(x) = 0, y'(0) = 1, y(0) = 0$

3. We study the differential equation $y''(x) + 5y'(x) + 4y(x) = e^{2x}$
(i) Solve the related homogeneous problem
(ii) Find a particular solution of the inhomogeous equation by susbtituting $y(x) = Ce^{zx}$, and determining the values for C and z.
(iii) Find the solution of the inhomegeneous equation satisfying $y(0) = 0$, $y'(0) = 0$.

Math Problems

4. Sketch the following surfaces:
(i) $f(x, y) = x$;
(ii) $f(x, y) = y$;
(iii) $f(x, y) = xy$.

5. Calculate the following partial derivatives
(i) $\frac{\partial}{\partial x} e^{-(x^2+y^2)}$
(ii) $\frac{\partial}{\partial y} \sin(x + x^2y^3)$
(iii) $\frac{\partial^2}{\partial x \partial y}(x^2 + xy^3)$
(iv) $\frac{\partial^2}{\partial y \partial x}(x^2 + xy^3)$

6. * Discuss and sketch the following:

(i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ ellipsoid,
(ii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ hyperboloid type 1,
(iii) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$ hyperboloid type 2,
(iv) $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$ cone,
(v) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = z$ elliptical paraboloid,
(vi) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = z$ hyperbolic paraboloid.

No assigned reading for next week: Coursework due