P201 Workshop 8, Week 8

Please follow the instructions of your supervisor regarding timing of these problems.

Math Practise

1. (i) Calculate $S_1 = 2 + 4 + 6 + ... + 100$.
   (ii) Calculate $S_2 = 1 + 2 + 4 + 8 + 16 + ... + 1024$.
   (iii) Calculate the binomials (n is an integer, use $0! = 1$)

$$\binom{4}{2}, \quad \binom{n}{n}, \quad \binom{n}{0}.$$

   (iv) Prove by induction that $\sum_{k=1}^{n}(2k-1) = n^2$.

2. (i) Calculate $S_1 = 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + ...$
   (ii) Calculate $S_2 = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + ...$
   (iii) Calculate (a and b arbitrary) $S_3 = 1 + ab + (ab)^2 + (ab)^3 + (ab)^4 + ...$ Which condition must be fulfilled for S_3 to converge?

3. Simplify the following expressions (a_k arbitrary)

$$\text{(i)} \quad S_1 = \sum_{k=0}^{n} a_k - \sum_{m=1}^{n+2} a_m,$$

$$\text{(ii)} \quad S_2 = \sum_{k=m}^{n} a_k - \sum_{k=m+1}^{n+2} a_{k-1}, \quad n > m$$

Math Problems

4. (i) Calculate the Taylor series of $f(x) = e^{2x}$ around $x = 0$.
   (ii) Calculate the n–th derivative ($n = 0, 1, 2, 3, ...$) of $f(x) = \sin(4x)$ at $x = 0$. Use the result to calculate the Taylor series of $\sin(x)$ around $x = 0$.
   (iii) Calculate the n–th derivative ($n = 0, 1, 2, 3, ...$) of $f(x) = \cos(3x)$ at $x = 0$. Use the result to calculate the Taylor series of $\cos(x)$ around $x = 0$.
   (iv) (Knowledge question) What is the Taylor series of $f(x) = -7 + x^2 + 1000x^6$ around $x = 0$?

5. * Prove Bernoulli's inequality $(1 + x)^n \geq 1 + nx, \quad x \geq -1$ for all integers n by induction $n \to n + 1$.

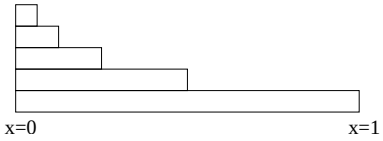
Physics

6. * A 'photon torpedo' is fired along the x–axis from $x = 0$ onto a looking–glass planet at $x = 1$ (right side). A part $0 < R < 1$ of the photon intensity is reflected to the left, a part $T = 1 - R$ of the intensity is transmitted to the right $x > 1$. The reflected part hits another looking–glass planet on the opposite side at $x = -1$, and again the part R of the torpedo gets reflected to the right (R^2 of the original intensity), and the part T is transmitted to the left side $x < -1$ (which is TR of the original intensity). The torpedo is thus bouncing back and forth between the two planets and gets weaker and weaker.

   (i) Make a sketch of this 'experiment'.

   (ii) Calculate the total transmitted photon 'torpedo' intensity T_r on the right ($x > 1$) and T_l on the left ($x < -1$) side after infinitely many reflections. Show that there is a 'sum rule' for the sum $T_r + T_l$ of both total transmitted photon 'torpedo' intensities.

7. ** Staircase



   (i) Calculate the centre-of-mass coordinate $\mathbf{r}_N = (X_N, Y_N)$ of a staircase of $N > 1$ rectangular blocks, each one on top of one with double size but the same height h and all blocks have equal mass density. (the figure shows the case $N = 5$).

   (ii) Calculate $\lim_{N\to\infty} X_N$.

Reading for next week: FM, Chap. 8