Nuclear and Particle Physics

Niels R Walet

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Last changed on March 23, 2010

Contents

Chapter 1

Introduction

In this course I shall discuss nuclear and particle physics on a somewhat phenomenological level. The mathematical sophistication shall be rather limited, with an emphasis on the physics and on symmetry aspects.

Course text:

W.E. Burcham and M. Jobes, *Nuclear and Particle Physics*, Addison Wesley Longman Ltd, Harlow, 1995.

Supplementary references

1. B.R. Martin and G. Shaw, *Particle Physics*, John Wiley and sons, Chicester, 1996. A solid book on particle physics, slightly more advanced than this course.

2. G.D. Coughlan and J.E. Dodd, *The ideas of particle physics*, Cambridge University Press, 1991. A more hand waving but more exciting introduction to particle physics. Reasonably up to date.

3. N.G. Cooper and G.B. West (eds.), *Particle Physics: A Los Alamos Primer*, Cambridge University Press, 1988. A bit less up to date, but very exciting and challenging book.

4. R. C. Fernow, *Introduction to experimental Particle Physics*, Cambridge University Press. 1986. A good source for experimental techniques and technology. A bit too advanced for the course.

5. F. Halzen and A.D. Martin, *Quarks and Leptons: An introductory Course in particle physics*, John Wiley and Sons, New York, 1984. A graduate level text book.

6. F.E. Close, *An introduction to Quarks and Partons*, Academic Press, London, 1979. Another highly recommendable graduate text.

7. The particle adventure: . A very nice–but slightly low level–introduction to particle physics.

Chapter 2

A history of particle physics

2.1 Nobel prices in particle physics

1903	BECQUEREL, ANTOINE HENRI, France, École Polytechnique, Paris, b. 1852, d. 1908: CURIE, PIERRE, France, cole municipale de physique et de chimie industrielles, (Municipal School of Industrial Physics and Chemistry), Paris, b. 1859, d. 1906; and his wife CURIE, MARIE, née SKLODOWSKA, France, b. 1867 (in Warsaw, Poland), d. 1934:	"in recognition of the extraordinary services he has rendered by his discovery of spontaneous radioactivity"; "in recognition of the extraordinary services they have rendered by their joint researches on the radiation phenomena discovered by Professor Henri Becquerel"
1922	BOHR, NIELS, Denmark, Copenhagen University, b. 1885, d. 1962:	"for his services in the investigation of the structure of atoms and of the radiation emanating from them"
1927	COMPTON, ARTHUR HOLLY, U.S.A., University of Chicago b. 1892, d. 1962: and WILSON, CHARLES THOMSON REES, Great Britain, Cambridge University, b. 1869 (in Glencorse, Scotland), d. 1959:	"for his discovery of the effect named after him"; "for his method of making the paths of electrically charged particles visible by condensation of vapour"
1932	HEISENBERG, WERNER, Germany, Leipzig University, b. 1901, d. 1976:	"for the creation of quantum mechanics, the application of which has, inter alia, led to the discovery of the allotropic forms of hydrogen"
	SCHRÖDINGER, ERWIN, Austria, Berlin University, Germany, b. 1887, d. 1961; and DIRAC, PAUL ADRIEN MAURICE, Great Britain, Cambridge University, b. 1902, d. 1984:	"for the discovery of new productive forms of atomic theory"
1935	CHADWICK, Sir JAMES, Great Britain, Liverpool University, b. 1891, d. 1974:	"for the discovery of the neutron"
1936	HESS, VICTOR FRANZ, Austria, Innsbruck University, b. 1883, d. 1964: ANDERSON, CARL DAVID, U.S.A., California Institute of Technology, Pasadena, CA, b. 1905, d. 1991:	"for his discovery of cosmic radiation"; and "for his discovery of the positron"
1938	FERMI, ENRICO, Italy, Rome University, b. 1901, d. 1954:	"for his demonstrations of the existence of new radioactive elements produced by neutron irradiation, and for his related discovery of nuclear reactions brought about by slow neutrons"
1939	LAWRENCE, ERNEST ORLANDO, U.S.A., University of California, Berkeley, CA, b. 1901, d. 1958:	"for the invention and development of the cyclotron and for results obtained with it, especially with regard to artificial radioactive elements"
1943	STERN, OTTO, U.S.A., Carnegie Institute of Technology, Pittsburg, PA, b. 1888 (in Sorau, then Germany), d. 1969:	"for his contribution to the development of the molecular ray method and his discovery of the magnetic moment of the proton"

1944	RABI, ISIDOR ISAAC, U.S.A., Columbia University, New York, NY, b. 1898, (in Rymanow, then Austria-Hungary) d. 1988:	"for his resonance method for recording the magnetic properties of atomic nuclei"
1945	PAULI, WOLFGANG, Austria, Princeton University, NJ, U.S.A., b. 1900, d. 1958:	"for the discovery of the Exclusion Principle, also called the Pauli Principle"
1948	BLACKETT, Lord PATRICK MAYNARD STUART, Great Britain, Victoria University, Manchester, b. 1897, d. 1974:	"for his development of the Wilson cloud chamber method, and his discoveries therewith in the fields of nuclear physics and cosmic radiation"
1949	YUKAWA, HIDEKI, Japan, Kyoto Imperial University and Columbia University, New York, NY, U.S.A., b. 1907, d. 1981:	"for his prediction of the existence of mesons on the basis of theoretical work on nuclear forces"
1950	POWELL, CECIL FRANK, Great Britain, Bristol University, b. 1903, d. 1969:	"for his development of the photographic method of studying nuclear processes and his discoveries regarding mesons made with this method"
1951	COCKCROFT, Sir JOHN DOUGLAS, Great Britain, Atomic Energy Research Establishment, Harwell, Didcot, Berks., b. 1897, d. 1967; and WALTON, ERNEST THOMAS SINTON, Ireland, Dublin University, b. 1903, d. 1995:	"for their pioneer work on the transmutation of atomic nuclei by artificially accelerated atomic particles"
1955	LAMB, WILLIS EUGENE, U.S.A., Stanford University, Stanford, CA, b. 1913: KUSCH, POLYKARP, U.S.A., Columbia University, New York, NY, b. 1911 (in Blankenburg, then Germany), d. 1993:	"for his discoveries concerning the fine structure of the hydrogen spectrum"; and "for his precision determination of the magnetic moment of the electron"
1957	YANG, CHEN NING, China, Institute for Advanced Study, Princeton, NJ, U.S.A., b. 1922; and LEE, TSUNG-DAO, China, Columbia University, New York, NY, U.S.A., b. 1926:	"for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles"
1959	SEGRÉ, EMILIO GINO, U.S.A., University of California, Berkeley, CA, b. 1905 (in Tivoli, Italy), d. 1989; and CHAMBERLAIN, OWEN, U.S.A., University of California, Berkeley, CA, b. 1920:	"for their discovery of the antiproton"
1960	GLASER, DONALD A., U.S.A., University of California, Berkeley, CA, b. 1926:	"for the invention of the bubble chamber"
1961	HOFSTADTER, ROBERT, U.S.A., Stanford University, Stanford, CA, b. 1915, d. 1990: MÖSSBAUER, RUDOLF LUDWIG, Germany, Technische Hochschule, Munich, and California Institute of Technology, Pasadena, CA, U.S.A., b. 1929:	"for his pioneering studies of electron scattering in atomic nuclei and for his thereby achieved discoveries concerning the stucture of the nucleons"; and "for his researches concerning the resonance absorption of gamma radiation and his discovery in this connection of the effect which bears his name"
1963	WIGNER, EUGENE P., U.S.A., Princeton University, Princeton, NJ, b. 1902 (in Budapest, Hungary), d. 1995: GOEPPERT-MAYER, MARIA, U.S.A., University of California, La Jolla, CA, b. 1906 (in Kattowitz, then Germany), d. 1972; and JENSEN, J. HANS D., Germany, University of Heidelberg, b. 1907, d. 1973:	"for his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles"; "for their discoveries concerning nuclear shell structure"

1965	TOMONAGA, SIN-ITIRO, Japan, Tokyo, University of Education, Tokyo, b. 1906, d. 1979; SCHWINGER, JULIAN, U.S.A., Harvard University, Cambridge, MA, b. 1918, d. 1994; and FEYNMAN, RICHARD P., U.S.A., California Institute of Technology, Pasadena, CA, b. 1918, d. 1988:	"for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles"
1967	BETHE, HANS ALBRECHT, U.S.A., Cornell University, Ithaca, NY, b. 1906 (in Strasbourg, then Germany):	"for his contributions to the theory of nuclear reactions, especially his discoveries concerning the energy production in stars"
1968	ALVAREZ, LUIS W., U.S.A., University of California, Berkeley, CA, b. 1911, d. 1988:	"for his decisive contributions to elementary particle physics, in particular the discovery of a large number of resonance states, made possible through his development of the technique of using hydrogen bubble chamber and data analysis"
1969	GELL-MANN, MURRAY, U.S.A., California Institute of Technology, Pasadena, CA, b. 1929:	"for his contributions and discoveries concerning the classification of elementary particles and their interactions"
1975	BOHR, AAGE, Denmark, Niels Bohr Institute, Copenhagen, b. 1922; MOTTELSON, BEN, Denmark, Nordita, Copenhagen, b. 1926 (in Chicago, U.S.A.); and RAINWATER, JAMES, U.S.A., Columbia University, New York, NY, b. 1917, d. 1986:	"for the discovery of the connection between collective motion and particle motion in atomic nuclei and the development of the theory of the structure of the atomic nucleus based on this connection"
1976	RICHTER, BURTON, U.S.A., Stanford Linear Accelerator Center, Stanford, CA, b. 1931; TING, SAMUEL C. C., U.S.A., Massachusetts Institute of Technology (MIT), Cambridge, MA, (European Center for Nuclear Research, Geneva, Switzerland), b. 1936:	"for their pioneering work in the discovery of a heavy elementary particle of a new kind"
1979	GLASHOW, SHELDON L., U.S.A., Lyman Laboratory, Harvard University, Cambridge, MA, b. 1932; SALAM, ABDUS, Pakistan, International Centre for Theoretical Physics, Trieste, and Imperial College of Science and Technology, London, Great Britain, b. 1926, d. 1996; and WEINBERG, STEVEN, U.S.A., Harvard University, Cambridge, MA, b. 1933:	"for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including inter alia the prediction of the weak neutral current"
1980	CRONIN, JAMES, W., U.S.A., University of Chicago, Chicago, IL, b. 1931; and FITCH, VAL L., U.S.A., Princeton University, Princeton, NJ, b. 1923:	"for the discovery of violations of fundamental symmetry principles in the decay of neutral K-mesons"
1983	CHANDRASEKHAR, SUBRAMANYAN, U.S.A., University of Chicago, Chicago, IL, b. 1910 (in Lahore, India), d. 1995: FOWLER, WILLIAM A., U.S.A., California Institute of Technology, Pasadena, CA, b. 1911, d. 1995:	"for his theoretical studies of the physical processes of importance to the structure and evolution of the stars" "for his theoretical and experimental studies of the nuclear reactions of importance in the formation of the chemical elements in the universe"

1984	RUBBIA, CARLO, Italy, CERN, Geneva, Switzerland, b. 1934; and VAN DER MEER, SIMON, the Netherlands, CERN, Geneva, Switzerland, b. 1925:	”for their decisive contributions to the large project, which led to the discovery of the field particles W and Z, communicators of weak interaction”
1988	LEDERMAN, LEON M., U.S.A., Fermi National Accelerator Laboratory, Batavia, IL, b. 1922; SCHWARTZ, MELVIN, U.S.A., Digital Pathways, Inc., Mountain View, CA, b. 1932; and STEINBERGER, JACK, U.S.A., CERN, Geneva, Switzerland, b. 1921 (in Bad Kissingen, FRG):	”for the neutrino beam method and the demonstration of the doublet structure of the leptons through the discovery of the muon neutrino”
1990	FRIEDMAN, JEROME I., U.S.A., Massachusetts Institute of Technology, Cambridge, MA, b. 1930; KENDALL, HENRY W., U.S.A., Massachusetts Institute of Technology, Cambridge, MA, b. 1926; and TAYLOR, RICHARD E., Canada, Stanford University, Stanford, CA, U.S.A., b. 1929:	”for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics”
1992	CHARPAK, GEORGES, France, École Supèrieure de Physique et Chimie, Paris and CERN, Geneva, Switzerland, b. 1924 (in Poland):	”for his invention and development of particle detectors, in particular the multiwire proportional chamber”
1995	 PERL, MARTIN L., U.S.A., Stanford University, Stanford, CA, U.S.A., b. 1927, REINES, FREDERICK, U.S.A., University of California at Irvine, Irvine, CA, U.S.A., b. 1918, d. 1998:	”for pioneering experimental contributions to lepton physics” ”for the discovery of the tau lepton” ”for the detection of the neutrino”

2.2 A time line

Particle Physics Time line

Year	Experiment	Theory
1927	β decay discovered	
1928		Paul Dirac: Wave equation for electron
1930		Wolfgang Pauli suggests existence of neutrino
1931	Positron discovered	
1931		Paul Dirac realises that positrons are part of his equation
1931	Chadwick discovers neutron	
1933/4		Fermi introduces theory for β decay
1933/4		Hideki Yukawa discusses nuclear binding in terms of pions
1937	μ discovered in cosmic rays	
1938	Baryon number conservation	
1946		μ is not Yukawa's particle
1947	π^+ discovered in cosmic rays	
1946-50		Tomonaga, Schwinger and Feynman develop QED
1948	First artificial π's	
1949	K^+ discovered	
1950	$\pi^0 \to \gamma\gamma$	
1951	"V-particles" Λ^0 and K^0	
1952	Δ: excited state of nucleon	
1954		Yang and Mills: Gauge theories
1956		Lee and Yang: Weak force might break parity!
1956	CS Wu and Ambler: Yes it does.	
1961		Eightfold way as organising principle
1962	ν_μ and ν_e	
1964		Quarks (Gell-man and Zweig) u, d, s
1964		Fourth quark suggested (c)
1965		Colour charge all particles are colour neutral!
1967		Glashow-Salam-Weinberg unification of electromagnetic and weak interactions. Predict Higgs boson.
1968-69	DIS at SLAC constituents of proton seen!	
1973		QCD as the theory of coloured interactions. Gluons.
1973		Asymptotic freedom
1974	J/ψ ($c\bar{c}$) meson	
1976	$D0$ meson ($\bar{u}c$) confirms theory.	
1976	τ lepton!	
1977	b (bottom quark). Where is top?	
1978	Parity violating neutral weak interaction seen	
1979	Gluon signature at PETRA	
1983	$W^\pm$ and Z^0 seen at CERN	
1989	SLAC suggests only three generations of (light!) neutrinos	
1995	t (top) at 175 GeV mass	
1997	New physics at HERA (200 GeV)	

2.3 Earliest stages

The early part of the 20th century saw the development of quantum theory and nuclear physics, of which particle physics detached itself around 1950. By the late 1920's one knew about the existence of the atomic nucleus, the electron and the proton. I shall start this history in 1927, the year in which the new quantum theory was introduced. In that year β decay was discovered as well: Some elements emit electrons with a continuous spectrum of energy. Energy conservation doesn't allow for this possibility (nuclear levels are discrete!). This led to the realisation, in 1929, by Wolfgang Pauli that one needs an additional particle to carry away the remaining energy and momentum. This was called a neutrino (small neutron) by Fermi, who also developed the first theoretical model of the process in 1933 for the decay of the neutron

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{2.1}$$

which had been discovered in 1931.

In 1928 Paul Dirac combined quantum mechanics and relativity in an equation for the electron. This equation had some more solutions than required, which were not well understood. Only in 1931 Dirac realised that these solutions are physical: they describe the positron, a positively charged electron, which is the *antiparticle* of the electron. This particle was discovered in the same year, and I would say that particle physics starts there.

2.4 fission and fusion

Fission of radioactive elements was already well established in the early part of the century, and activation by neutrons, to generate more unstable isotopes, was investigated before fission of natural isotopes was seen. The inverse process, fusion, was understood somewhat later, and Niels Bohr developed a model describing the nucleus as a fluid drop. This model - the collective model - was further developed by his son Aage Bohr and Ben Mottelson. A very different model of the nucleus, the shell model, was designed by Maria Goeppert-Mayer and Hans Jensen in 1952, concentrating on individual nucleons. The dichotomy between a description as individual particles and as a collective whole characterises much of "low-energy" nuclear physics.

2.5 Low-energy nuclear physics

The field of low-energy nuclear physics, which concentrates mainly on structure of and low-energy reaction on nuclei, has become one of the smaller parts of nuclear physics (apart from in the UK). Notable results have included better understanding of the nuclear medium, high-spin physics, super deformation and halo nuclei. Current experimental interest is in those nuclei near the "drip lines" which are of astrophysical importance, as well as of other interest.

2.6 Medium-energy nuclear physics

Medium energy nuclear physics is interested in the response of a nucleus to probes at such energies that we can no longer consider nucleons to be elementary particles. Most modern experiments are done by electron scattering, and concentrate on the role of QCD (see below) in nuclei, the structure of mesons in nuclei and other complicated questions.

2.7 high-energy nuclear physics

This is not a very well-defined field, since particle physicists are also working here. It is mainly concerned with ultra-relativistic scattering of nuclei from each other, addressing questions about the quark-gluon plasma. It should be nuclear physics, since we consider "dirty" systems of many particles, which are what nuclear physicists are good at.

2.8 Mesons, leptons and neutrinos

In 1934 Yukawa introduces a new particle, the pion (π), which can be used to describe nuclear binding. He estimates it's mass at 200 electron masses. In 1937 such a particle is first seen in cosmic rays. It is later realised that it interacts too weakly to be the pion and is actually a lepton (electron-like particle) called the μ. The π is found (in cosmic rays) and is the progenitor of the μ's that were seen before:

$$\pi^+ \to \mu^+ + \nu_\mu \tag{2.2}$$

The next year artificial pions are produced in an accelerator, and in 1950 the neutral pion is found,

$$\pi^0 \to \gamma\gamma. \tag{2.3}$$

This is an example of the conservation of electric charge. Already in 1938 Stuckelberg had found that there are other conserved quantities: the number of baryons (n and p and ...) is also conserved!

After a serious break in the work during the latter part of WWII, activity resumed again. The theory of electrons and positrons interacting through the electromagnetic field (photons) was tackled seriously, and with important contributions of (amongst others) Tomonaga, Schwinger and Feynman was developed into a highly accurate tool to describe hyperfine structure.

Experimental activity also resumed. Cosmic rays still provided an important source of extremely energetic particles, and in 1947 a "strange" particle (K^+ was discovered through its very peculiar decay pattern. Balloon experiments led to additional discoveries: So-called V particles were found, which were neutral particles, identified as the Λ^0 and K^0. It was realised that a new conserved quantity had been found. It was called strangeness.

The technological development around WWII led to an explosion in the use of accelerators, and more and more particles were found. A few of the important ones are the antiproton, which was first seen in 1955, and the Δ, a very peculiar excited state of the nucleon, that comes in four charge states Δ^{++}, Δ^+, Δ^0, Δ^-.

Theory was develop-ping rapidly as well. A few highlights: In 1954 Yang and Mills develop the concept of gauged Yang-Mills fields. It looked like a mathematical game at the time, but it proved to be the key tool in developing what is now called "the standard model".

In 1956 Yang and Lee make the revolutionary suggestion that parity is not necessarily conserved in the weak interactions. In the same year "madam" CS Wu and Alder show experimentally that this is true: God is weakly left-handed!

In 1957 Schwinger, Bludman and Glashow suggest that all weak interactions (radioactive decay) are mediated by the charged bosons $W^\pm$. In 1961 Gell-Mann and Ne'eman introduce the "eightfold way": a mathematical taxonomy to organise the particle zoo.

2.9 The sub-structure of the nucleon (QCD)

In 1964 Gell-Mann and Zweig introduce the idea of quarks: particles with spin 1/2 and fractional charges. They are called up, down and strange and have charges $2/3, -1/3, -1/3$ times the electron charge.

Since it was found (in 1962) that electrons and muons are each accompanied by their own neutrino, it is proposed to organise the quarks in multiplets as well:

$$\begin{array}{lll} e & \nu_e & (u,d) \\ \mu & \nu_\mu & (s,c) \end{array} \tag{2.4}$$

This requires a fourth quark, which is called charm.

In 1965 Greenberg, Han and Nambu explain why we can't see quarks: quarks carry colour charge, and all observe particles must have colour charge 0. Mesons have a quark and an antiquark, and baryons must be build from three quarks through its peculiar symmetry.

The first evidence of quarks is found (1969) in an experiment at SLAC, where small pips inside the proton are seen. This gives additional impetus to develop a theory that incorporates some of the ideas already found: this is called QCD. It is shown that even though quarks and gluons (the building blocks of the theory) exist, they cannot be created as free particles. At very high energies (very short distances) it is found that they behave more and more like real free particles. This explains the SLAC experiment, and is called asymptotic freedom.

The J/ψ meson is discovered in 1974, and proves to be the $c\bar{c}$ bound state. Other mesons are discovered (D0, $\bar{u}c$) and agree with QCD.

In 1976 a third lepton, a heavy electron, is discovered (τ). This was unexpected! A matching quark (b for bottom or beauty) is found in 1977. Where is its partner, the top? It will only be found in 1995, and has a mass of 175 GeV/c^2 (similar to a lead nucleus...)! Together with the conclusion that there are no further light neutrinos (and one might hope no quarks and charged leptons) this closes a chapter in particle physics.

2.10 The $W^{\pm}$ and Z bosons

On the other side a electro-weak interaction is developed by Weinberg and Salam. A few years later 't Hooft shows that it is a well-posed theory. This predicts the existence of three extremely heavy bosons that mediate the weak force: the Z^0 and the $W^{\pm}$. These have been found in 1983. There is one more particle predicted by these theories: the Higgs particle. Must be very heavy!

2.11 GUTS, Supersymmetry, Supergravity

This is not the end of the story. The standard model is surprisingly inelegant, and contains way to many parameters for theorists to be happy. There is a dark mass problem in astrophysics – most of the mass in the universe is not seen! This all leads to the idea of an underlying theory. Many different ideas have been developed, but experiment will have the last word! It might already be getting some signals: researchers at DESY see a new signal in a region of particle that are 200 GeV heavy – it might be noise, but it could well be significant!

There are several ideas floating around: one is the grand-unified theory, where we try to combine all the disparate forces in nature in one big theoretical frame. Not unrelated is the idea of supersymmetries: For every "boson" we have a "fermion". There are some indications that such theories may actually be able to make useful predictions.

2.12 Extraterrestrial particle physics

One of the problems is that it is difficult to see how e can actually build a microscope that can look a a small enough scale, i.e., how we can build an accelerator that will be able to accelerate particles to high enough energies? The answer is simple – and has been more or less the same through the years: Look at the cosmos. Processes on an astrophysical scale can have amazing energies.

2.12.1 Balloon experiments

One of the most used techniques is to use balloons to send up some instrumentation. Once the atmosphere is no longer the perturbing factor it normally is, one can then try to detect interesting physics. A problem is the relatively limited payload that can be carried by a balloon.

2.12.2 Ground based systems

These days people concentrate on those rare, extremely high energy processes (of about 10^{29} eV), where the effect of the atmosphere actually help detection. The trick is to look at showers of (lower-energy) particles created when such a high-energy particle travels through the earth's atmosphere.

2.12.3 Dark matter

One of the interesting cosmological questions is whether we live in an open or closed universe. From various measurements we seem to get conflicting indications about the mass density of (parts of) the universe. It seems that the ration of luminous to non-luminous matter is rather small. Where is all that "dark mass": Mini-Jupiter's, small planetoids, dust, or new particles....

2.12.4 (Solar) Neutrinos

The neutrino is a very interesting particle. Even though we believe that we understand the nuclear physics of the sun, the number of neutrinos emitted from the sun seems to anomalously small. Unfortunately this is very hard to measure, and one needs quite a few different experiments to disentangle the physics behind these processes. Such experiments are coming on line in the next few years. These can also look at neutrinos coming from other astrophysical sources, such as supernovas, and enhance our understanding of those processes. Current indications from Kamiokande are that neutrinos do have mass, but oscillation problems still need to be resolved.

Chapter 3

Experimental tools

In this chapter we shall concentrate on the experimental tools used in nuclear and particle physics. Mainly the present ones, but it is hard to avoid discussing some of the history.

3.1 Accelerators

3.1.1 Resolving power

Both nuclear and particle physics experiments are typically performed at accelerators, where particles are accelerated to extremely high energies, in most cases relativistic (i.e., $v \approx c$). To understand why this happens we need to look at the rôle the accelerators play. Accelerators are nothing but extremely big microscopes. At ultrarelativistic energies it doesn't really matter what the mass of the particle is, its energy only depends on the momentum:

$$E = h\nu = \sqrt{m^2c^4 + \boldsymbol{p}^2c^2} \approx pc \tag{3.1}$$

from which we conclude that

$$\lambda = \frac{c}{\nu} = \frac{h}{p}. \tag{3.2}$$

The typical resolving power of a microscope is about the size of one wave-length, λ. For an an ultrarelativistic particle this implies an energy of

$$E = pc = h\frac{c}{\lambda} \tag{3.3}$$

You may not immediately appreciate the enormous scale of these energies. An energy of 1 TeV ($= 10^{12}$eV)

Table 3.1: Size and energy-scale for various objects

particle	scale	energy
atom	10^{-10}m	2 keV
nucleus	10^{-14}m	20 MeV
nucleon	10^{-15}m	200 MeV
quark?	$< 10^{-18}$m	>200 GeV

is 3×10^{-7} J, which is the same as the kinetic energy of a 1g particle moving at 1.7 cm/s. And that for particles that are of submicroscopic size! We shall thus have to push these particles very hard indeed to gain such energies. In order to push these particles we need a handle to grasp hold of. The best one we know of is to use charged particles, since these can be accelerated with a combination of electric and magnetic fields – it is easy to get the necessary power as well.

3.1.2 Types

We can distinguish accelerators in two ways. One is whether the particles are accelerated along straight lines or along (approximate) circles. The other distinction is whether we used a DC (or slowly varying AC) voltage, or whether we use radio-frequency AC voltage, as is the case in most modern accelerators.

3.1.3 DC fields

Acceleration in a DC field is rather straightforward: If we have two plates with a potential V between them, and release a particle near the plate at lower potential it will be accelerated to an energy $\frac{1}{2}mv^2 = eV$. This was the original technique that got Cockroft and Wolton their Nobel prize.

van der Graaff generator

A better system is the tandem van der Graaff generator, even though this technique is slowly becoming obsolete in nuclear physics (technological applications are still very common). The idea is to use a (non-conducting) rubber belt to transfer charge to a collector in the middle of the machine, which can be used to build up sizable (20 MV) potentials. By sending in negatively charged ions, which are stripped of (a large number of) their electrons in the middle of the machine we can use this potential twice. This is the mechanism used in part of the Daresbury machine.

Other linear accelerators

Linear accelerators (called Linacs) are mainly used for electrons. The idea is to use a microwave or radio frequency field to accelerate the electrons through a number of connected cavities (DC fields of the desired strength are just impossible to maintain). A disadvantage of this system is that electrons can only be accelerated in tiny bunches, in small parts of the time. This so-called "duty-cycle", which is small (less than a percent) makes these machines not so beloved. It is also hard to use a linac in colliding beam mode (see below).

There are two basic setups for a linac. The original one is to use elements of different length with a fast oscillating (RF) field between the different elements, designed so that it takes exactly one period of the field to traverse each element. Matched acceleration only takes place for particles traversing the gaps when the field is almost maximal, actually sightly before maximal is OK as well. This leads to bunches coming out.

More modern electron accelerators are build using microwave cavities, where standing microwaves are generated. Such a standing wave can be thought of as one wave moving with the electron, and another moving the other wave. If we start of with relativistic electrons, $v \approx c$, this wave accelerates the electrons. This method requires less power than the one above.

Cyclotron

The original design for a circular accelerator dates back to the 1930's, and is called a cyclotron. Like all circular accelerators it is based on the fact that a charged particle (charge qe) in a magnetic field B with velocity v moves in a circle of radius r, more precisely

$$qvB = \frac{\gamma m v^2}{r}, \tag{3.4}$$

where γm is the relativistic mass, $\gamma = (1-\beta^2)^{-1/2}$, $\beta = v/c$. A cyclotron consists of two metal "D"-rings, in which the particles are shielded from electric fields, and an electric field is applied between the two rings, changing sign for each half-revolution. This field then accelerates the particles.

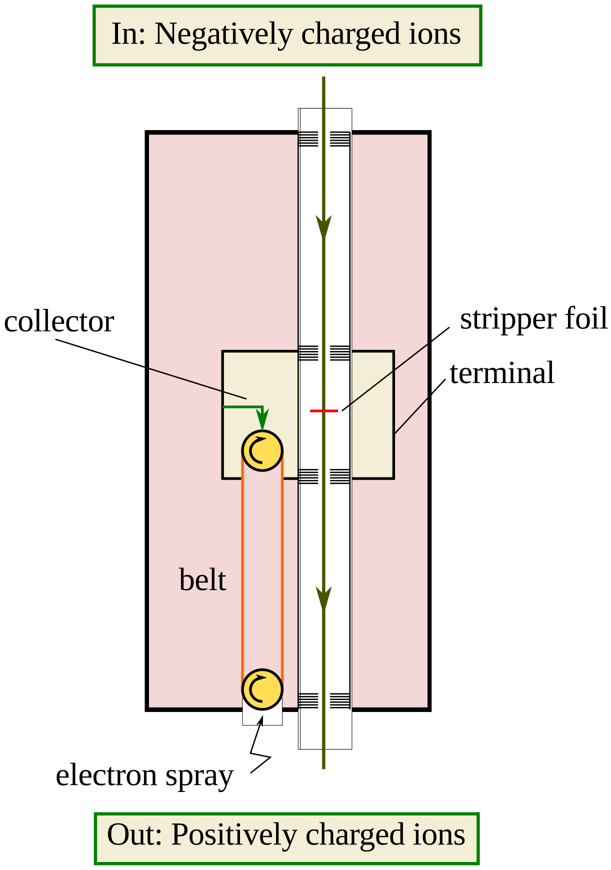

Figure 3.1: A sketch of a tandem van der Graaff generator

The field has to change with a frequency equal to the angular velocity,

$$f = \frac{\omega}{2\pi} = \frac{v}{2\pi r} = \frac{qB}{2\pi\gamma m}. \tag{3.5}$$

For non-relativistic particles, where $\gamma \approx 1$, we can thus run a cyclotron at constant frequency, 15.25 MHz/T for protons. Since we extract the particles at the largest radius possible, we can determine the velocity and thus the energy,

$$E = \gamma mc^2 = [(qBRc)^2 + m^2c^4]^{1/2} \tag{3.6}$$

Synchroton

The shear size of a cyclotron that accelerates particles to 100 GeV or more would be outrageous. For that reason a different type of accelerator is used for higher energy, the so-called synchroton where the particles are accelerated in a circle of constant diameter.

In a circular accelerator (also called synchroton), see Fig. 3.5, we have a set of magnetic elements that bend the beam of charged into an almost circular shape, and empty regions in between those elements where a high frequency electro-magnetic field accelerates the particles to ever higher energies. The particles make many passes through the accelerator, at every increasing momentum. This makes critical timing requirements on the accelerating fields, they cannot remain constant.

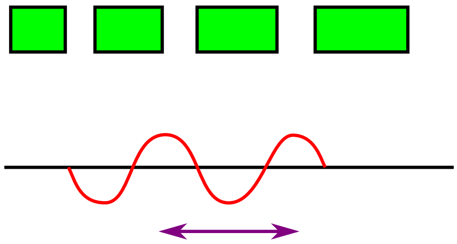

Figure 3.2: A sketch of a linac

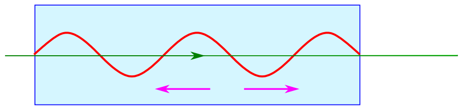

Figure 3.3: Acceleration by a standing wave

Using the equations given above, we find that

$$f = \frac{qB}{2\pi\gamma m} = \frac{qBc^2}{2\pi E} = \frac{qBc^2}{2\pi(m^2c^4 + q^2B^2R^2c^2)^{1/2}} \tag{3.7}$$

For very high energy this goes over to

$$f = \frac{c}{2\pi R}, \qquad E = qBRc, \tag{3.8}$$

so we need to keep the frequency constant whilst increasing the magnetic field. In between the bending elements we insert (here and there) microwave cavities that accelerate the particles, which leads to bunching, i.e., particles travel with the top of the field.

So what determines the size of the ring and its maximal energy? There are two key factors:
As you know, a free particle does not move in a circle. It needs to be accelerated to do that. The magnetic elements take care of that, but an accelerated charge radiates – That is why there are synchroton lines at Daresbury! The amount of energy lost through radiation in one pass through the ring is given by (all

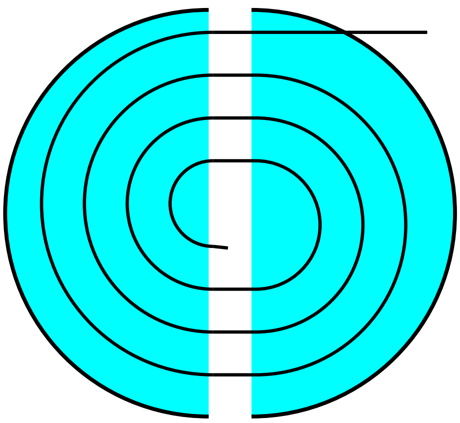

Figure 3.4: A sketch of a cyclotron

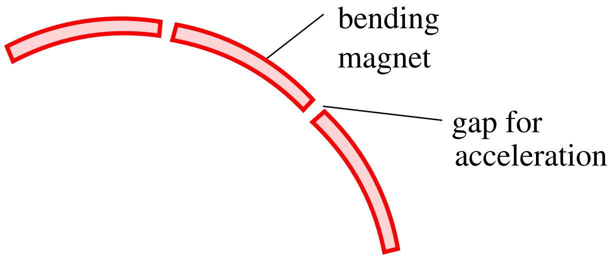

Figure 3.5: A sketch of a synchroton

quantities in SI units)

$$\Delta E = \frac{4\pi}{3\epsilon_0} \frac{q^2 \beta^3 \gamma^4}{R} \tag{3.9}$$

with $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$, and R is the radius of the accelerator in meters. In most cases $v \approx c$, and we can replace β by 1. We can also use one of the charges to re-express the energy-loss in eV:

$$\Delta E \approx \frac{4\pi}{3\epsilon_0} \frac{q\gamma^4}{R} \Delta E \approx \frac{4\pi}{3\epsilon_0} \frac{q}{R} \left(\frac{E}{mc^2}\right)^4 . \tag{3.10}$$

Thus the amount of energy lost is proportional to the fourth power of the relativistic energy, $E = \gamma mc^2$. For an electron at 1 TeV energy γ is

$$\gamma_e = \frac{E}{m_e c^2} = \frac{10^{12}}{511 \times 10^3} = 1.9 \times 10^6 \tag{3.11}$$

and for a proton at the same energy

$$\gamma_p = \frac{E}{m_p c^2} = \frac{10^{12}}{939 \times 10^6} = 1.1 \times 10^3 \tag{3.12}$$

This means that a proton looses a lot less energy than an electron (the fourth power in the expression shows the difference to be 10^{12}!). Let us take the radius of the ring to be 5 km (large, but not extremely so). We find the results listed in table 3.1.3.

Table 3.2: Energy loss for a proton or electron in a synchroton of radius 5km

proton	E	ΔE
	1 GeV	1.5×10^{-11} eV
	10 GeV	1.5×10^{-7} eV
	100 GeV	1.5×10^{-3} eV
	1000 GeV	1.5×10^{1} eV
electron	E	ΔE
	1 GeV	2.2×10^{2} eV
	10 GeV	2.2 MeV
	100 GeV	22 GeV
	1000 GeV	2.2×10^{15} GeV

The other key factor is the maximal magnetic field. From the standard expression for the centrifugal force we find that the radius R for a relativistic particle is related to it's momentum (when expressed in

GeV/c) by

$$p = 0.3BR \tag{3.13}$$

For a standard magnet the maximal field that can be reached is about 1T, for a superconducting one 5T. A particle moving at $p = 1\text{TeV/c} = 1000\text{GeV/c}$ requires a radius of

Table 3.3: Radius R of an synchroton for given magnetic fields and momenta.

B	p	R
1 T	1 GeV/c	3.3 m
	10 GeV/c	33 m
	100 GeV/c	330 m
	1000 GeV/c	3.3 km
5 T	1 GeV/c	0.66 m
	10 GeV/c	6.6 m
	100 GeV/c	66 m
	1000 GeV/c	660 m

3.2 Targets

There are two ways to make the necessary collisions with the accelerated beam: Fixed target and colliding beams.

In fixed target mode the accelerated beam hits a target which is fixed in the laboratory. Relativistic kinematics tells us that if a particle in the beam collides with a particle in the target, their centre-of-mass (four) momentum is conserved. The only energy remaining for the *reaction* is the relative energy (or energy within the cm frame). This can be expressed as

$$E_{CM} = \left[m_b^2 c^4 + m_t^2 c^4 + 2m_t c^2 E_L\right]^{1/2} \tag{3.14}$$

where m_b is the mass of a beam particle, m_t is the mass of a target particle and E_L is the beam energy as measured in the laboratory. as we increase E_L we can ignore the first tow terms in the square root and we find that

$$E_{CM} \approx \sqrt{2m_t c^2 E_L}, \tag{3.15}$$

and thus the centre-of-mass energy only increases as the square root of the lab energy!

In the case of colliding beams we use the fact that we have (say) an electron beam moving one way, and a positron beam going in the opposite direction. Since the centre of mass is at rest, we have the full energy of both beams available,

$$E_{CM} = 2E_L. \tag{3.16}$$

This grows linearly with lab energy, so that a factor two increase in the beam energy also gives a factor two increase in the available energy to produce new particles! We would only have gained a factor $\sqrt{2}$ for the case of a fixed target. This is the reason that almost all modern facilities are colliding beams.

3.3 The main experimental facilities

Let me first list a couple of facilities with there energies, and then discuss the facilities one-by-one.

Table 3.4: Fixed target facilities, and their beam energies

accelerator	facility	particle	energy
KEK	Tokyo	p	12 GeV
SLAC	Stanford	e^-	25GeV
PS	CERN p	28 GeV	
AGS	BNL	p 32 GeV	
SPS	CERN	p 250 GeV	
Tevatron II	FNL	p	1000 GeV

Table 3.5: Colliding beam facilities, and their beam energies

accelerator	facility	particle & energy (in GeV)
CESR	Cornell	$e^+(6) + e^-(6)$
PEP	Stanford	$e^+(15) + e^-(15)$
Tristan	KEK	$e^+(32) + e^-(32)$
SLC	Stanford	$e^+(50) + e^-(50)$
LEP	CERN	$e^+(60) + e^-(60)$
Spp̄S	CERN	$p(450) + \bar{p}(450)$
Tevatron I	FNL	$p(1000) + \bar{p}(1000)$
LHC	CERN	$e^-(50) + p(8000)$
		$p(8000) + \bar{p}(8000)$

3.3.1 SLAC (B factory, Babar)

Stanford Linear Accelerator Center, located just south of San Francisco, is the longest linear accelerator in the world. It accelerates electrons and positrons down its 2-mile length to various targets, rings and detectors at its end. The PEP ring shown is being rebuilt for the B factory, which will study some of the mysteries of antimatter using B mesons. Related physics will be done at Cornell with CESR and in Japan with KEK.

3.3.2 Fermilab (D0 and CDF)

Fermi National Accelerator Laboratory, a high-energy physics laboratory, named after particle physicist pioneer Enrico Fermi, is located 30 miles west of Chicago. It is the home of the world's most powerful particle accelerator, the Tevatron, which was used to discover the top quark.

3.3.3 CERN (LEP and LHC)

CERN (European Laboratory for Particle Physics) is an international laboratory where the W and Z bosons were discovered. CERN is the birthplace of the World-Wide Web. The Large Hadron Collider (see below) will search for Higgs bosons and other new fundamental particles and forces.

3.3.4 Brookhaven (RHIC)

Brookhaven National Laboratory (BNL) is located on Long Island, New York. Charm quark was discovered there, simultaneously with SLAC. The main ring (RHIC) is 0.6 km in radius.

3.3.5 Cornell (CESR)

The Cornell Electron-Positron Storage Ring (CESR) is an electron-positron collider with a circumference of 768 meters, located 12 meters below the ground at Cornell University campus. It is capable of producing collisions between electrons and their anti-particles, positrons, with centre-of-mass energies between 9

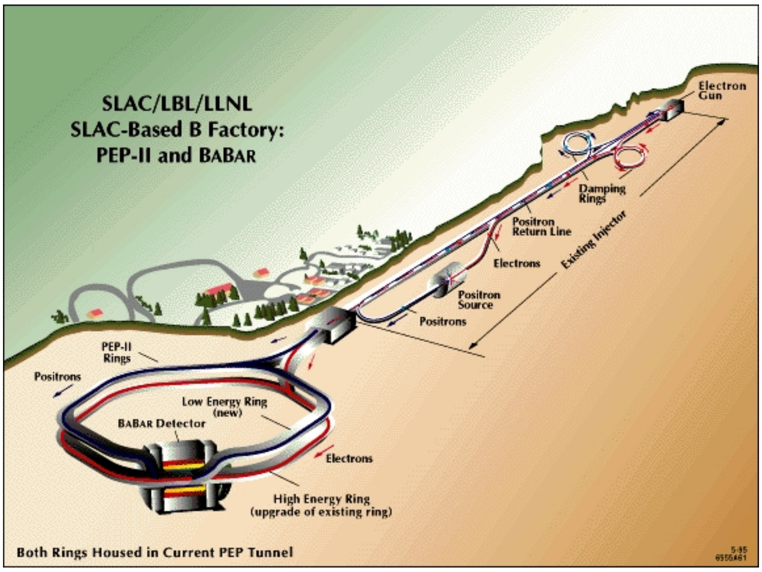

Figure 3.6: A picture of SLAC

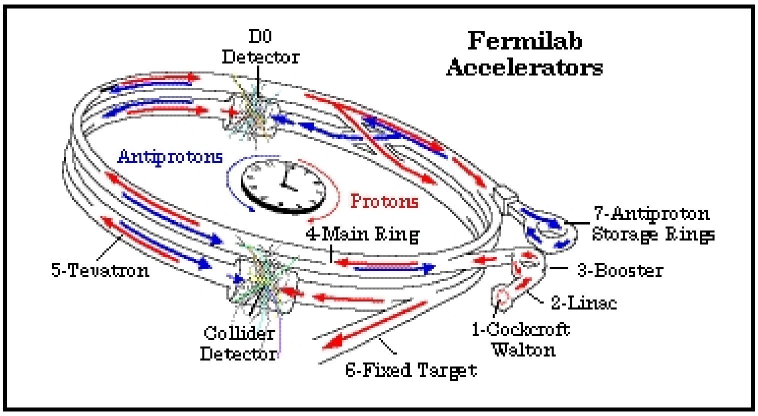

Figure 3.7: A picture of Fermilab

and 12 GeV. The products of these collisions are studied with a detection apparatus, called the CLEO detector.

3.3.6 DESY (Hera and Petra)

The DESY laboratory, located in Hamburg, Germany, discovered the gluon at the PETRA accelerator. DESY consists of two accelerators: HERA and PETRA. These accelerators collide electrons and protons.

3.3.7 KEK (tristan)

The KEK laboratory, in Japan, was originally established for the purpose of promoting experimental studies on elementary particles. A 12 GeV proton synchrotron was constructed as the first major facility. Since its commissioning in 1976, the proton synchrotron played an important role in boosting experimental activities in Japan and thus laid the foundation of the next step of KEK's high energy physics program, a 30 GeV electron-positron colliding-beam accelerator called TRISTAN.

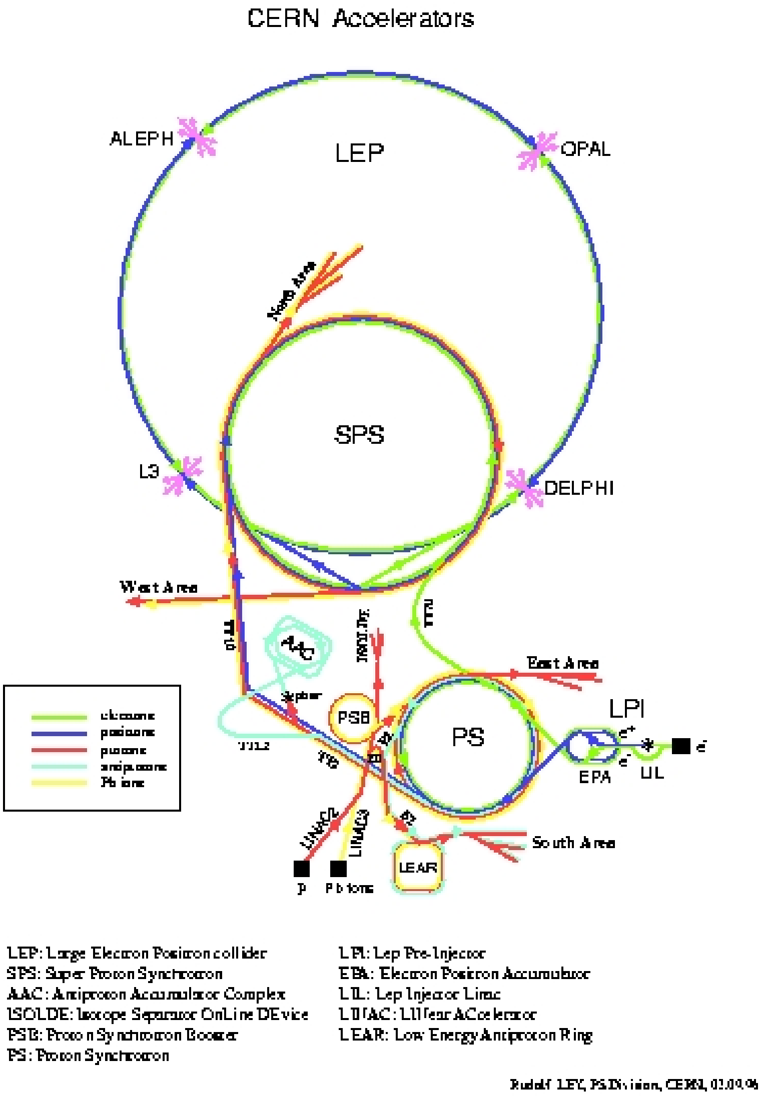

Figure 3.8: A picture of CERN

3.3.8 IHEP

Institute for High-Energy Physics, in the People's Republic of China, performs detailed studies of the tau lepton and charm quark.

3.4 Detectors

Detectors are used for various measurements on the physical processes occurring in particle physics. The most important of those are

- To identify particles.
- To measure positions.
- To measure time differences.

Figure 3.9: A picture of Brookhaven National Lab

- To measure momentum.
- To measure energy.

Let me now go over some of the different pieces of machinery used to perform such measurements

3.4.1 Scintillation counters

This is based on the fact that charged particles traversing solids excite the electrons in such materials. In some solids light is then emitted. This light can be collected and amplified by photomultipliers. This technique has a very fast time response, of about 200 ps. For this reason one uses scintillators as "trigger". This means that a pulse from the scintillator is used to say that data should now be accepted from the other pieces of equipment.

Another use is to measure time-of-flight. When one uses a pair of scintillation detectors, one can measure the time difference for a particle hitting both of them, thus determining a time difference and velocity. This is only useful for slow particles, where v differs from c by a reasonable amount.

3.4.2 Proportional/Drift Chamber

Once again we use charged particles to excite electrons. We now use a gas, where the electrons get liberated. We then use the fact that these electrons drift along electric field lines to collect them on wires. If we have many such wires, we can see where the electrons were produced, and thus measure positions with an accuracy of 500 μm or less.

3.4.3 Semiconductor detectors

Using modern techniques we can etch very fine strips on semiconductors. We can easily have multiple layers of strips running along different directions as well. These can be used to measure position (a hit in a certain set of strips). Typical resolutions are 5 μm. A problem with such detectors is so-called radiation damage, due to the harsh environment in which they are operated.

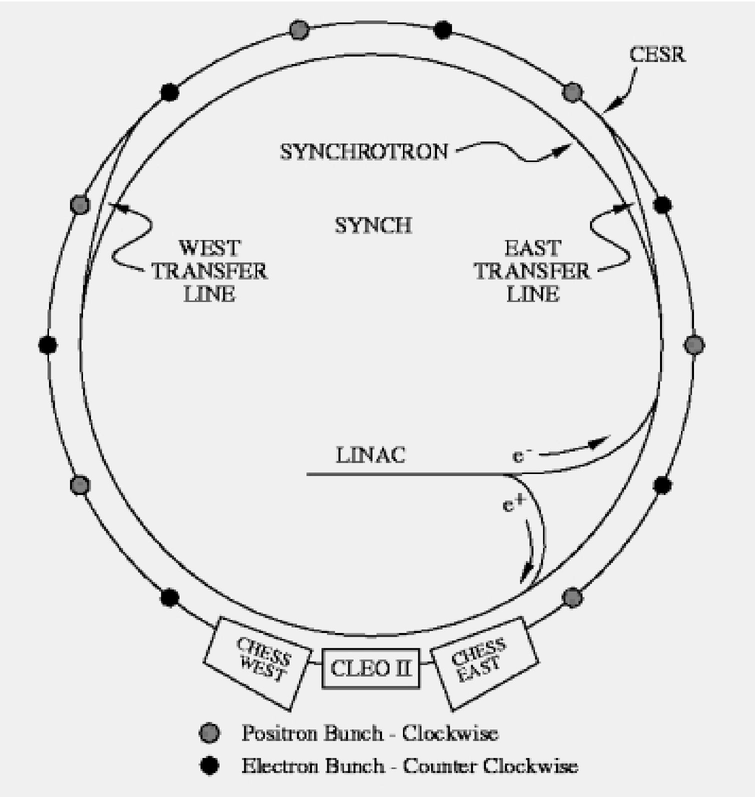

Figure 3.10: A picture of the Cornell accelerator

3.4.4 Spectrometer

One uses a magnet with a position sensitive detector at the end to bend the track of charged particles, and determine the radius of the circular orbit. This radius is related to the momentum of the particles.

3.4.5 Čerenkov Counters

These are based on the analogue of a supersonic boom. When a particles velocity is higher than the speed of light in medium, $v > c/n$, where n is the index of refraction we get a shock wave. As can be seen in Fig. 3.14a) for slow motion the light emitted by a particle travels faster than the particle (the circles denote how far the light has travelled). On the other hand, when the particle moves faster than the speed of light, we get a linear wave-front propagating through the material, as sketched in Fig. 3.14b. The angle of this wave front is related to the speed of the particles, by $\cos\theta = \frac{1}{\beta n}$. Measuring this angle allows us to determine speed (a problem here is the small number of photons emitted). This technique is extremely useful for threshold counters, because if we see any light, we know that the velocity of particles is larger than c/n.

3.4.6 Transition radiation

3.4.7 Calorimeters

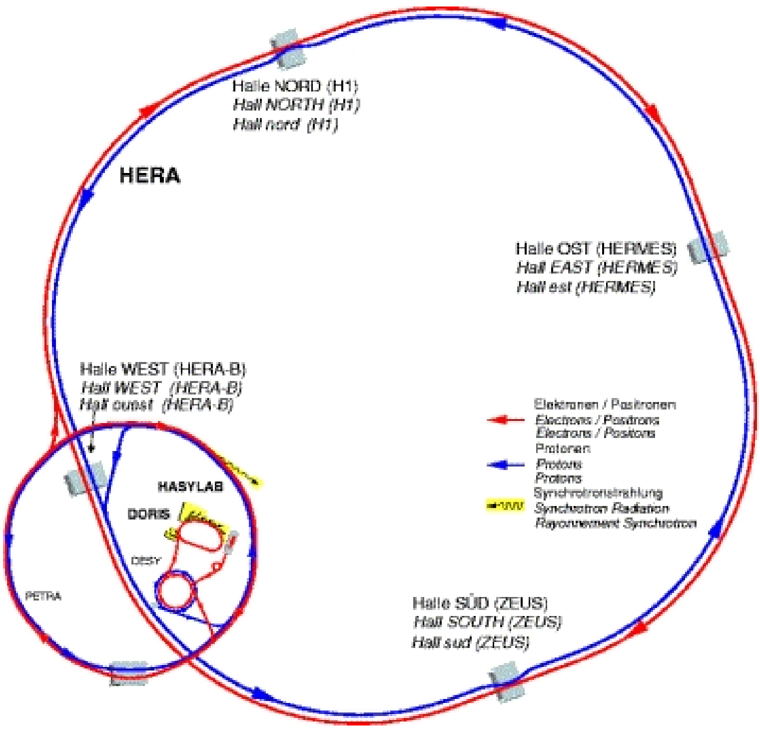

Figure 3.11: A picture of HERA

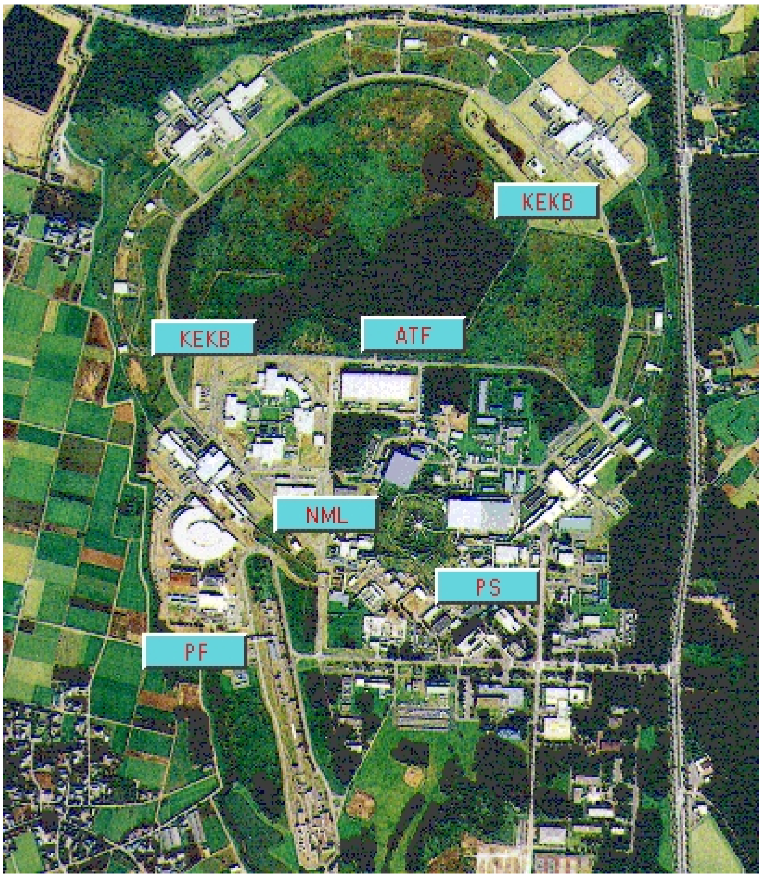

Figure 3.12: A picture of KEK

Figure 3.13: A picture of IHEP

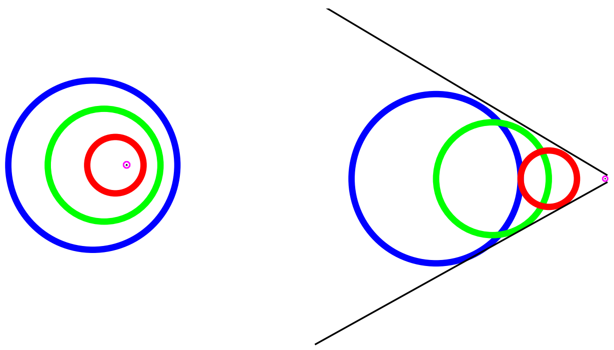

Figure 3.14: Čerenkov radiation

Chapter 4

Nuclear Masses

4.1 Experimental facts

1. Each nucleus has a (positive) charge Ze, and integer number times the elementary charge e. This follows from the fact that atoms are neutral!

2. Nuclei of identical charge come in different masses, all approximate multiples of the "nucleon mass". (Nucleon is the generic term for a neutron or proton, which have almost the same mass, $m_p = 938.272\mathrm{MeV}/c^2$, $m_n = \mathrm{MeV}/c^2$.) Masses can easily be determined by analysing nuclei in a *mass spectrograph* which can be used to determine the relation between the charge Z (the number of protons, we believe) vs. the mass.

Nuclei of identical charge (chemical type) but different mass are called isotopes. Nuclei of approximately the same mass, but different chemical type, are called isobars.

4.1.1 mass spectrograph

A mass spectrograph is a combination of a bending magnet, and an electrostatic device (to be completed).

4.2 Interpretation

We conclude that the nucleus of mass $m \approx Am_N$ contains Z positively charged nucleons (protons) and $N = A - Z$ neutral nucleons (neutrons). These particles are bound together by the "nuclear force", which changes the mass below that of free particles. We shall typically write ${}^{A}\mathrm{El}$ for an element of chemical type El, which determines Z, containing A nucleons.

4.3 Deeper analysis of nuclear masses

To analyse the masses even better we use the atomic mass unit (amu), which is 1/12th of the mass of the neutral carbon atom,

$$1\ \mathrm{amu} = \frac{1}{12} m_{^{12}\mathrm{C}}. \tag{4.1}$$

This can easily be converted to SI units by some chemistry. One mole of ${}^{12}\mathrm{C}$ weighs 0.012 kg, and contains Avogadro's number particles, thus

$$1\ \mathrm{amu} = \frac{0.001}{N_A}\ \mathrm{kg} = 1.66054 \times 10^{-27}\ \mathrm{kg} = 931.494\mathrm{MeV}/c^2. \tag{4.2}$$

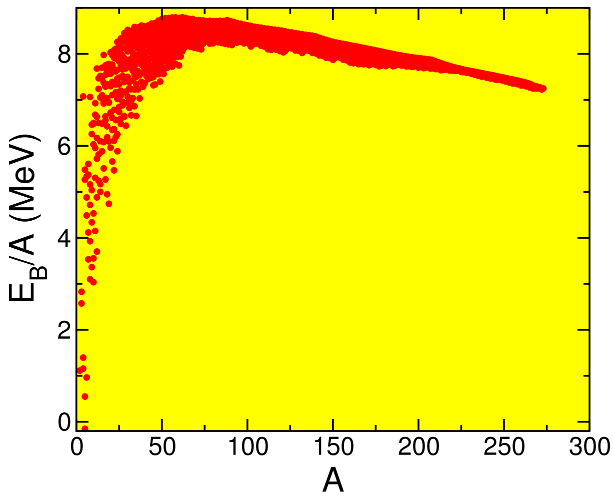

Figure 4.1: B/A versus A

The quantity of most interest in understanding the mass is the binding energy, defined for a neutral atom as the difference between the mass of a nucleus and the mass of its constituents,

$$B(A, Z) = ZM_H c^2 + (A - Z)M_n c^2 - M(A, Z)c^2. \tag{4.3}$$

With this choice a system is bound when $B > 0$, when the mass of the nucleus is lower than the mass of its constituents. Let us first look at this quantity per nucleon as a function of A, see Fig. 4.1

This seems to show that to a reasonable degree of approximation the mass is a function of A alone, and furthermore, that it approaches a constant. This is called nuclear saturation. This agrees with experiment, which suggests that the radius of a nucleus scales with the 1/3rd power of A,

$$R_{\mathrm{RMS}} \approx 1.1 A^{1/3}\ \mathrm{fm}. \tag{4.4}$$

This is consistent with the saturation hypothesis made by Gamov in the 30's:

As A increases the volume per nucleon remains constant.

For a spherical nucleus of radius R we get the condition

$$\frac{4}{3}\pi R^3 = AV_1, \tag{4.5}$$

or

$$R = \left(\frac{V_1 3}{4\pi}\right)^{1/3} A^{1/3}. \tag{4.6}$$

From which we conclude that

$$V_1 = 5.5\ \mathrm{fm}^3 \tag{4.7}$$

4.4 Nuclear mass formula

There is more structure in Fig. 4.1 than just a simple linear dependence on A. A naive analysis suggests that the following terms should play a rôle:

1. Bulk energy: This is the term studied above, and saturation implies that the energy is proportional to $B_{\mathrm{bulk}} = \alpha A$.

Table 4.1: Fit of masses to Eq. (4.8)

.

parameter	value
α	15.36 MeV
β	16.32 MeV
γ	90.45 MeV
ϵ	0.6928 MeV

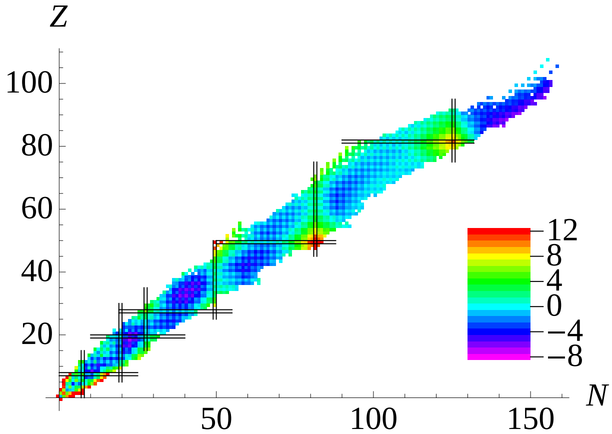

Figure 4.2: Difference between fitted binding energies and experimental values, as a function of N and Z.

2. Surface energy: Nucleons at the surface of the nuclear sphere have less neighbours, and should feel less attraction. Since the surface area goes with R^2, we find $B_{\text{surface}} = -\beta A$.

3. Pauli or symmetry energy: nucleons are fermions (will be discussed later). That means that they cannot occupy the same states, thus reducing the binding. This is found to be proportional to $B_{\text{symm}} = -\gamma(N/2 - Z/2)^2/A^2$.

4. Coulomb energy: protons are charges and they repel. The average distance between is related to the radius of the nucleus, the number of interaction is roughly Z^2 (or $Z(Z-1)$). We have to include the term $B_{\text{Coul}} = -\epsilon Z^2/A$.

Taking all this together we fit the formula

$$B(A, Z) = \alpha A - \beta A^{2/3} - \gamma(A/2 - Z)^2 A^{-1} - \epsilon Z^2 A^{-1/3} \tag{4.8}$$

to all know nuclear binding energies with $A \geq 16$ (the formula is not so good for light nuclei). The fit results are given in table 4.1.

In Fig. 4.3 we show how well this fit works. There remains a certain amount of structure, see below, as well as a strong difference between neighbouring nuclei. This is due to the superfluid nature of nuclear material: nucleons of opposite momenta tend to anti-align their spins, thus gaining energy. The solution is to add a pairing term to the binding energy,

$$B_{\text{pair}} = \begin{cases} A^{-1/2} & \text{for } N \text{ odd, } Z \text{ odd} \\ -A^{-1/2} & \text{for } N \text{ even, } Z \text{ even} \end{cases} \tag{4.9}$$

Table 4.2: Fit of masses to Eq. (4.10)

parameter	value
α	15.36 MeV
β	16.32 MeV
γ	90.46 MeV
δ	11.32 MeV
ϵ	0.6929 MeV

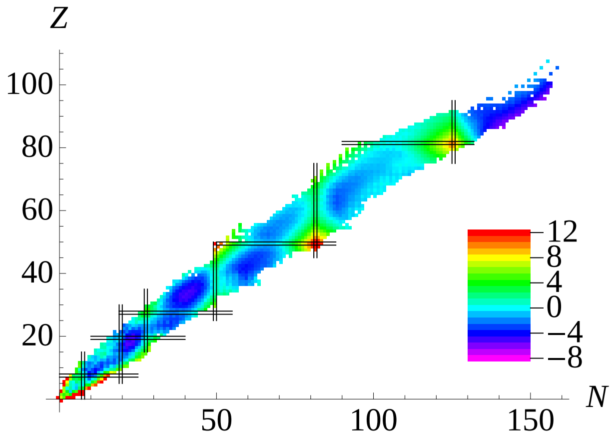

Figure 4.3: Difference between fitted binding energies and experimental values, as a function of N and Z.

The results including this term are significantly better, even though all other parameters remain at the same position, see Table 4.2. Taking all this together we fit the formula

$$B(A,Z) = \alpha A - \beta A^{2/3} - \gamma (A/2 - Z)^2 A^{-1} - \delta B_{\text{pair}}(A,Z) - \epsilon Z^2 A^{-1/3} \tag{4.10}$$

4.5 Stability of nuclei

In figure 4.5 we have colour coded the nuclei of a given mass $A = N + Z$ by their mass, red for those of lowest mass through to magenta for those of highest mass. We can see that typically the nuclei that are most stable for fixed A have more neutrons than protons, more so for large A increases than for low A. This is the "neutron excess".

4.5.1 β decay

If we look at fixed nucleon number A, we can see that the masses vary strongly,

It is known that a free neutron is not a stable particle, it actually decays by emission of an electron and an antineutrino,

$$n \rightarrow p + e^- + \bar{\nu}_e. \tag{4.11}$$

The reason that this reaction can take place is that it is endothermic, $m_n c^2 > m_p c^2 + m_e c^2$. (Here we assume that the neutrino has no mass.) The degree of allowance of such a reaction is usually expressed in a Q value, the amount of energy released in such a reaction,

$$Q = m_n c^2 - m_p c^2 - m_e c^2 = 939.6 - 938.3 - 0.5 = 0.8 \text{ MeV}. \tag{4.12}$$

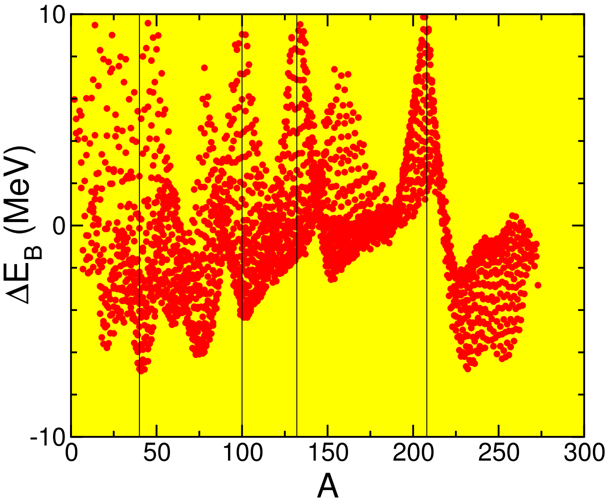

Figure 4.4: B/A versus A, mass formula subtracted

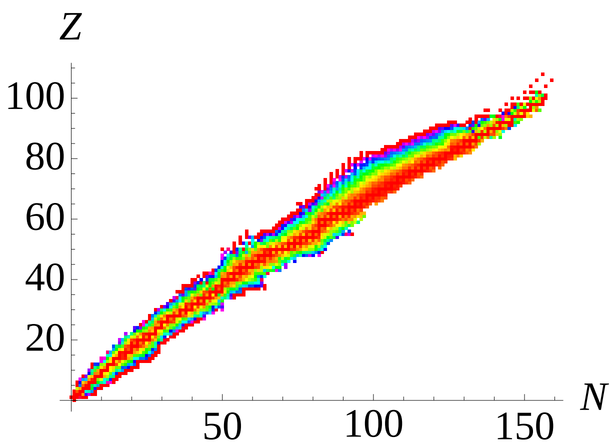

Figure 4.5: The valley of stability

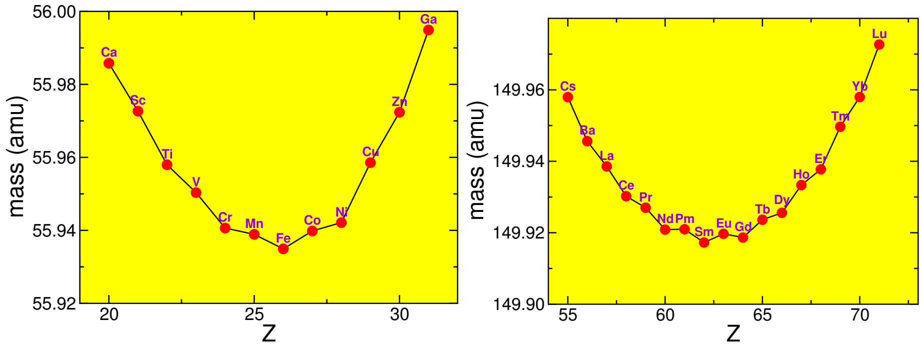

Figure 4.6: A cross section through the mass table for fixed A. To the left, $A = 56$, and to the right, $A = 150$.

Generically it is found that two reaction may take place, depending on the balance of masses. Either a neutron "β decays" as sketched above, or we have the inverse reaction

$$p \to n + e^+ + \nu_e. \tag{4.13}$$

For historical reason the electron or positron emitted in such a process is called a β particle. Thus in β^- decay of a nucleus, a nucleus of Z protons and N neutrons turns into one of $Z+1$ protons and $N-1$ neutrons (moving towards the right in Fig. 4.6). In β^+ decay the nucleus moves to the left. Since in that figure I am using atomic masses, the Q factor is

$$\begin{aligned} Q_{\beta^-} &= M(A,Z)c^2 - M(A,Z+1)c^2, \\ Q_{\beta^-} &= M(A,Z)c^2 - M(A,Z-1)c^2 - 2m_e c^2. \end{aligned} \tag{4.14}$$

The double electron mass contribution in this last equation because the atom looses one electron, as well as emits a positron with has the same mass as the electron.

In similar ways we can study the fact whether reactions where a single nucleon (neutron or proton) is emitted, as well as those where more complicated objects, such as Helium nuclei (α particles) are emitted. I shall return to such processed later, but let us note the Q values,

$$\begin{aligned} \text{neutron emission} \quad & Q = (M(A,Z) - M(A-1,Z) - m_n)c^2, \\ \text{proton emission} \quad & Q = (M(A,Z) - M(A-1,Z-1) - M(1,1))c^2, \\ \alpha \text{ emission} \quad & Q = (M(A,Z) - M(A-4,Z-2) - M(4,2))c^2, \\ \text{break up} \quad & Q = (M(A,Z) - M(A-A_1,Z-Z_1) - M(A_1,Z_1))c^2. \end{aligned} \tag{4.15}$$

4.6 properties of nuclear states

Nuclei are quantum systems, and as such must be described by a quantum Hamiltonian. Fortunately nuclear energies are much smaller than masses, so that a description in terms of non-relativistic quantum mechanics is possible. Such a description is not totally trivial since we have to deal with quantum systems containing many particles. Rather then solving such complicated systems, we often resort to models. We can establish, on rather general grounds, that nuclei are

4.6.1 quantum numbers

As in any quantum system there are many quantum states in each nucleus. These are labelled by their quantum numbers, which, as will be shown later, originate in symmetries of the underlying Hamiltonian, or rather the underlying physics.

angular momentum

One of the key invariances of the laws of physics is rotational invariance, i.e., physics is independent of the direction you are looking at. This leads to the introduction of a vector angular momentum operator,

$$\hat{\boldsymbol{L}} = \hat{\boldsymbol{r}} \times \hat{\boldsymbol{p}}, \tag{4.16}$$

which generates rotations. As we shall see later quantum states are not necessarily invariant under the rotation, but transform in a well-defined way. The three operators $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ satisfy a rather intriguing structure,

$$[\hat{L}_x, \hat{L}_y] \equiv \hat{L}_x\hat{L}_y - \hat{L}_y\hat{L}_x = i\hbar\hat{L}_z, \tag{4.17}$$

and the same for q cyclic permutation of indices ($xyz \to yzx$ or zxy). This shows that we cannot determine all three components simultaneously in a quantum state. One normally only calculates the length of the

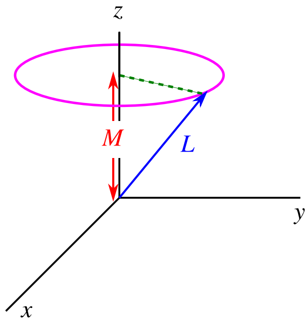

Figure 4.7: A pictorial representation of the "quantum precession" of an angular momentum of fixed length L and projection M.

angular momentum vector, and its projection on the z axis,

$$\begin{aligned} \hat{\boldsymbol{L}}^2 \phi_{LM} &= \hbar^2 L(L+1)\phi_{LM}, \\ \hat{L}_z \phi_{LM} &= \hbar L_z \phi_{LM}. \end{aligned} \tag{4.18}$$

It can be shown that L is a non-negative integer, and M is an integer satisfying $|M| < L$, i.e., the projection is always smaller than or equal to the length, a rather simple statement in classical mechanics.

The standard, albeit slightly simplified, picture of this process is that of a fixed length angular momentum precessing about the z axis, keeping the projection fixed, as shown in Fig. 4.7.

The energy of a quantum state is independent of the M quantum number, since the physics is independent of the orientation of L in space (unless we apply a magnetic field that breaks this symmetry). We just find multiplets of $2L+1$ states with the same energy and value of L, differing only in M.

Unfortunately the story does not end here. Like electrons, protons and neutron have a spin, i.e., we can use a magnetic field to separate nucleons with spin up from those with spin down. Spins are like orbital angular momenta in many aspects, we can write three operators $\hat{\mathbf{S}}$ that satisfy the same relation as the $\hat{L}$'s, but we find that

$$\hat{\mathbf{S}}^2 \phi_{S,S_z} = \hbar^2 \frac{3}{4} \phi_{S,S_z}, \tag{4.19}$$

i.e., the length of the spin is $1/2$, with projections $\pm 1/2$.

Spins will be shown to be coupled to orbital angular momentum to total angular momentum J,

$$\hat{\boldsymbol{J}} = \hat{\boldsymbol{L}} + \hat{\mathbf{S}}, \tag{4.20}$$

and we shall specify the quantum state by L, S, J and J_z. This can be explained pictorially as in Fig. 4.8. There we show how, for fixed length J the spin and orbital angular momentum precess about the vector $\boldsymbol{J}$, which in its turn precesses about the z-axis. It is easy to see that if $vecL$ and $\boldsymbol{S}$ are fully aligned we have $J = L + S$, and if they are anti-aligned $J = |L - S|$. A deeper quantum analysis shows that this is the way the quantum number work. If the angular momentum quantum numbers of the states being coupled are L and S, the length of the resultant vector J can be

$$J = |L-S|, |L-S|+2, \ldots, L+S. \tag{4.21}$$

We have now discussed the angular momentum quantum number for a single particle. For a nucleus which in principle is made up from many particles, we have to add all these angular momenta together until we get something called the total angular momentum. Since the total angular momentum of a single particle is half-integral (why?), the total angular momentum of a nucleus is integer for even A, and half-integer for odd A.

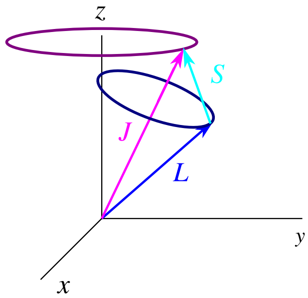

Figure 4.8: A pictorial representation of the vector addition of spin and orbital angular momentum.

Parity

Another symmetry of the wave function is parity. If we change $\boldsymbol{r} \to -\boldsymbol{r}$, i.e., mirror space, the laws of physics are invariant. Since we can do this operation twice and get back where we started from, any eigenvalue of this operation must be ± 1, usually denoted as $\Pi = \pm$. It can be shown that for a particle with orbital angular momentum L, $\Pi = (-1)^L$. The parity of many particles is just the product of the individual parities.

isotopic spin (Isobaric spin, isospin)

The most complicated symmetry in nuclear physics is isospin. In contrast to the symmetries above this is not exact, but only approximate. The first clue of this symmetry come from the proton and neutron masses, $m_n = \mathrm{MeV}/c^2$ and $m_p = \mathrm{MeV}/c^2$, and their very similar behaviour in nuclei. Remember that the dominant binding terms only depended on the number of nucleons, not on what type of nucleons we are dealing with.

All of this leads to the assumption of another abstract quantity, called isospin, which describes a new symmetry of nature. We assume that both neutrons and protons are manifestation of one single particle, the nucleon, with isospin down or up, respectively. We shall have to see whether this makes sense by looking in more detail at the nuclear physics. We propose the identification

$$Q = (I_z + 1/2)e, \tag{4.22}$$

where I_z is the z projection of the vectorial quantity called isospin. Apart from the neutron-proton mass difference, isospin symmetry in nuclei is definitely broken by the Coulomb force, which acts on protons but not on neutrons. We shall argue that the nuclear force, that couples to the "nucleon charge" rather than electric charge, respects this symmetry. What we shall do is look at a few nuclei where we can study both a nucleus and its mirror image under the exchange of protons and neutrons. One example are the nuclei ^{7}He and ^{7}B (2 protons and 5 neutrons, $I_z = -3/2$ vs. 5 protons and 2 neutrons, $I_z = 3/2$) and ^{7}Li and ^{7}B (3 protons and 4 neutrons, $I_z = -1/2$ vs. 4 protons and 3 neutrons, $I_z = 1/2$), as sketched in Fig. 4.9.

We note there the great similarity between the pairs of mirror nuclei. Of even more importance is the fact that the $3/2^-;3/2$ level occurs at the same energy in all four nuclei, suggestion that we can define these states as an "isospin multiplet", the same state just differing by I_z.

4.6.2 deuteron

Let us think of the deuteron (initially) as a state with $L = 0$, $J = 1$, $S = 1$, usually denoted as $^3S_1$ (S means $L = 0$, the 3 denotes $S = 1$, i.e., three possible spin orientations, and the subscript 1 the value of J). Let us model the nuclear force as a three dimensional square well with radius R. The Schrödinger equation for

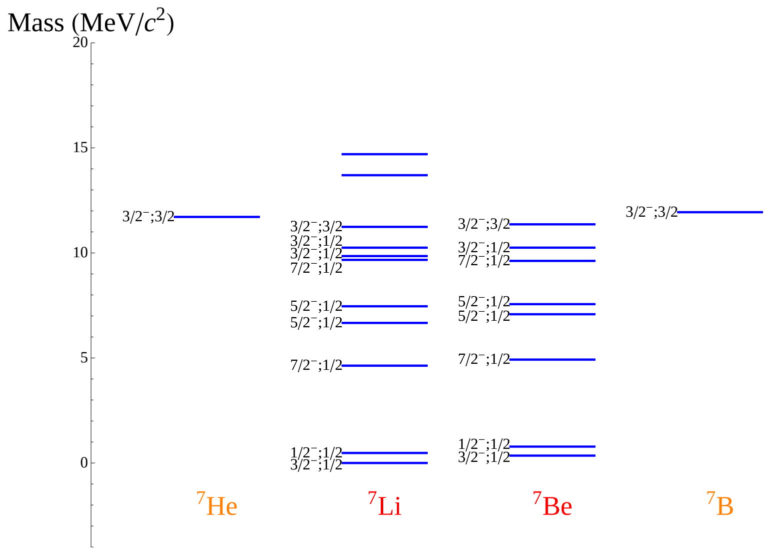

Figure 4.9: The spectrum of the nuclei with $A = 7$. The label of each state is J, parity, isospin. The zeroes of energy were determined by the relative nuclear masses.

the spherically symmetric S state is (work in radial coordinates)

$$-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\left(\frac{d}{dr}r^2\frac{d}{dr}R(r)\right) + V(r)R(r) = ER(r). \tag{4.23}$$

Here $V(r)$ is the potential, and μ is the reduced mass,

$$\mu = \frac{m_n m_p}{m_n + m_p}, \tag{4.24}$$

which arises from working in the relative coordinate only. It is easier to work with $u(r) = rR(r)$, which satisfies the condition

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}u(r) + V(r)u(r) = Eu(r), \tag{4.25}$$

as well as $u(0) = 0$. The equation in the interior

$$-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}u(r)V_0 u(r) = Eu(r), \qquad u(0) = 0 \tag{4.26}$$

has as solution

$$u = A\sin\kappa r, \qquad \kappa = \sqrt{\frac{2\mu}{\hbar^2}(V_0 + E)}. \tag{4.27}$$

Outside the well we find the standard damped exponential,

$$u = B\exp(kr), \qquad k = \sqrt{\frac{2\mu}{\hbar^2}(-E)}. \tag{4.28}$$

Matching derivatives at the boundary we find

$$-\cot\kappa R = \frac{k}{\kappa} = \sqrt{\frac{-E}{V_0 + E}}. \tag{4.29}$$

We shall now make the assumption that $|E| \ll V_0$, which will prove true. Then we find

$$\kappa \approx \sqrt{\frac{2\mu}{\hbar^2} V_0}, \qquad \cot \kappa R \approx 0. \tag{4.30}$$

Since it is known from experiment that the deuteron has only one bound state at energy -2.224573 ± 0.000002 MeV, we see that $\kappa R \approx \pi/2$! Substituting κ we see that

$$V_0 R^2 = \frac{\pi^2 \hbar^2}{8\mu}. \tag{4.31}$$

If we take $V_0 = 30$ MeV, we find $R = 1.83$ fm.

We can orient the spins of neutron and protons in a magnetic field, i.e., we find that there is an energy

$$E_{magn} = \mu_N \mu \boldsymbol{S} \cdot \boldsymbol{B}. \tag{4.32}$$

(The units for this expression is the so-called nuclear magneton, $\mu_N = \frac{e\hbar}{2m_p}$.) Experimentally we know that

$$\mu_n = -1.91315 \pm 0.00007 \mu_N \qquad \mu_p = 2.79271 \pm 0.00002 \mu_N \tag{4.33}$$

If we compare the measured value for the deuteron, $\mu_d = 0.857411 \pm 0.000019 \mu_N$, with the sum of protons and neutrons (spins aligned), we see that $\mu_p + \mu_n = 0.857956 \pm 0.00007 \mu_N$. The close agreement suggest that the spin assignment is largely OK; the small difference means that our answer cannot be the whole story: we need other components in the wave function.

We know that an S state is spherically symmetric and cannot have a quadrupole moment, i.e., it does not have a preferred axis of orientation in an electric field. It is known that the deuteron has a positive quadrupole moment of $0.29 e^2$ fm^2, corresponding to an elongation of the charge distribution along the spin axis.

From this we conclude that the deuteron wave function carries a small (7%) component of the $^3D_1$ state (D: $L = 2$). We shall discuss later on what this means for the nuclear force.

4.6.3 Scattering of nucleons

We shall concentrate on scattering in an $L = 0$ state only, further formalism just gets too complicated. For definiteness I shall just look at the scattering in the $^3S_1$ channel, and the $^1S_0$ one. (These are also called the triplet and singlet channels.)

(*not discussed this year! Needs some filling in.*)

4.6.4 Nuclear Forces

Having learnt this much about nuclei, what can we say about the nuclear force, the attraction that holds nuclei together? First of all, from Rutherford's old experiments on α particle scattering from nuclei, one can learn that the range of these forces is a few fm.

From the fact that nuclei saturate, and are bound, we would then naively build up a picture of a potential that is strongly repulsive at short distances, and shows some mild attraction at a range of 1-2 fm, somewhat like sketched in Fig. 4.10.

Here we assume, that just as the Coulomb force can be derived from a potential that only depends on the size of r,

$$V(r) = \frac{q_1 q_2}{4\pi\epsilon_0 r}, \tag{4.34}$$

the nuclear force depends only on r as well. This is the simplest way to construct a rotationally invariant energy. For particles with spin other possibilities arise as well (e.g., $\hat{\boldsymbol{S}} \cdot \boldsymbol{r}$) so how can we see what the nuclear force is really like?

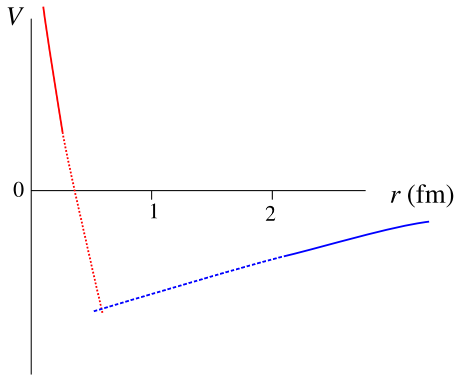

Figure 4.10: Possible form for the internucleon potential, repulsive at short distances, and attractive at large distances.

Since we have taken the force to connect pairs of particles, we can just study the interaction of two nucleons, by looking both at the bound states (there is only one), and at scattering, where we study how a nucleon gets deflected when it scatters of another nucleon. Let us first look at the deuteron, the bound state of a proton and a neutron. The quantum numbers of its ground state are $J^\pi = 1^+$, $I = 0$. A little bit of additional analysis shows that this is a state with $S = 1$, and $L = 0$ or 2. Naively one would expect a lowest state $S = 0$, $L = 0$ (which must have $I =$ for symmetry reasons not discussed here). So what can we read of about the nuclear force from this result?

We conclude the following:

1. The nuclear force in the S waves is attractive.
2. Nuclear binding is caused by the tensor force.
3. The nuclear force is isospin symmetric (i.e., it is independent of the direction of isospin).

Chapter 5

Nuclear models

There are two important classed of nuclear models: single particle and microscopic models, that concentrate on the individual nucleons and their interactions, and collective models, where we just model the nucleus as a collective of nucleons, often a nuclear fluid drop.

Microscopic models need to take into account the Pauli principle, which states that no two nucleons can occupy the same quantum state. This is due to the Fermi-Dirac statistics of spin 1/2 particles, which states that the wave function is antisymmetric under interchange of any two particles

5.1 Nuclear shell model

The simplest of the single particle models is the nuclear shell model. It is based on the observation that the nuclear mass formula, which describes the nuclear masses quite well on average, fails for certain "magic numbers", i.e., for neutron number $N = 20, 28, 50, 82, 126$ and proton number $Z = 20, 28, 50, 82$, as indicated if Fig. xxxx. These nuclei are much more strongly bound than the mass formula predicts, especially for the doubly magic cases, i.e., when N and Z are both magic. Further analysis suggests that this is due to a shell structure, as has been seen in atomic physics.

5.1.1 Mechanism that causes shell structure

So what causes the shell structure? In atoms it is the Coulomb force of the heavy nucleus that forces the electrons to occupy certain orbitals. This can be seen as an external agent. In nuclei no such external force exits, so we have to find a different mechanism.

The solution, and the reason the idea of shell structure in nuclei is such a counter-intuitive notion, is both elegant and simple. Consider a single nucleon in a nucleus. Within this nuclear fluid we can consider the interactions of each of the nucleons with the one we have singled out. All of these nucleons move rather quickly through this fluid, leading to the fact that our nucleons only sees the average effects of the attraction of all the other ones. This leads to us replacing, to first approximation, this effect by an average nuclear potential, as sketched in Fig. 5.1.

Thus the idea is that the shell structure is caused by the average field of all the other nucleons, a very elegant but rather surprising notion!

5.1.2 Modelling the shell structure

Whereas in atomic physics we solve the Coulomb force problem to get the shell structure, we expect that in nuclei the potential is more attractive in the centre, where the density is highest, and less attractive near the surface. There is no reason why the attraction should diverge anywhere, and we expect the potential to be finite everywhere. One potential that satisfies these criteria, and can be solved analytically, is the

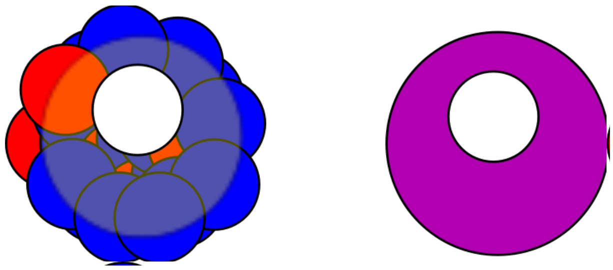

Figure 5.1: A sketch of the averaging approximation

Harmonic oscillator potential. Let us use that as a first model, and solve

$$-\frac{\hbar^2}{2m}\Delta\psi(\mathbf{r}) + \tfrac{1}{2}m\omega^2 r^2\psi(\mathbf{r}) = E\psi(\mathbf{r}). \tag{5.1}$$

The easiest way to solve this equation is to realise that, since $r^2 = x^2 + y^2 + z^2$, the Hamiltonian is actually a sum of an x, y and z harmonic oscillator, and the eigenvalues are the sum of those three oscillators,

$$E_{n_x n_y n_z} = (n_x + n_y + n_z + 3/2)\hbar\omega. \tag{5.2}$$

The great disadvantage of this form is that it ignores the rotational invariance of the potential. If we separate the Schrödinger equation in radial coordinates as

$$R_{nl}(r)Y_{LM}(\theta,\phi) \tag{5.3}$$

with Y the spherical harmonics, we find

$$-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}R(r)\right) + \frac{\hbar^2 L(L+1)}{r^2}R(r) + \tfrac{1}{2}m\omega^2 r^2 R(r) = E_{nL}R(r). \tag{5.4}$$

In this case it can be shown that

$$E_{nL} = (2n + L + 3/2)\hbar\omega, \tag{5.5}$$

and L is the orbital angular momentum of the state. We use the standard, so-called spectroscopic, notation of $s, p, d, f, g, h, i, j, \ldots$ for $L = 0, 1, 2, \ldots$.

In the left hand side of Fig. 5.2 we have list the number harmonic oscillator quanta in each set of shells. We have made use of the fact that in the real potentials state of the same number of quanta but different L are no longer degenerate, but there are groups of shells with big energy gaps between them. This cannot predict the magic numbers beyond 20, and we need to find a different mechanism. There is one already known for atoms, which is the so-called spin orbit splitting. This means that the degeneracy in the total angular momentum ($j = L \pm 1/2$) is lifted by an energy term that splits the aligned from anti-aligned case. This is shown schematically in the right of the figure, where we label the states by n, l and j. The gaps between the groups of shells are in reality much larger than the spacing within one shell, making the binding-energy of a closed-shell nucleus much lower than that of its neighbours.

5.1.3 evidence for shell structure

Evidence for the shell structure can be seen in two ways:
1- By looking at nuclear reactions that add a nucleon or remove a nucleon from a closed shell nucleus. The most sensitive of these are electron knockout reactions, where an electron comes in and an electron and a proton or neutron escapes, usually denoted as $(e,e'p)$ $(e,e'n)$ reactions. In those we see clear evidence of peaks at the single particle energies.

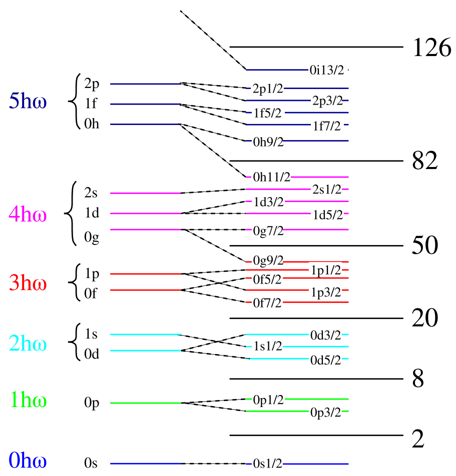

Figure 5.2: A schematic representation of the shell structure in nuclei.

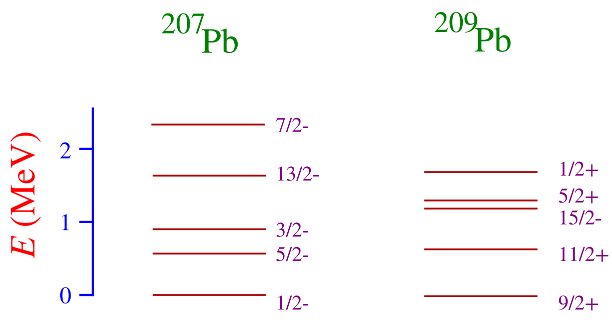

Figure 5.3: The spectra for the one-neutron hole nucleus ^{207}Pb and the one-particle nucleus ^{209}Pb.

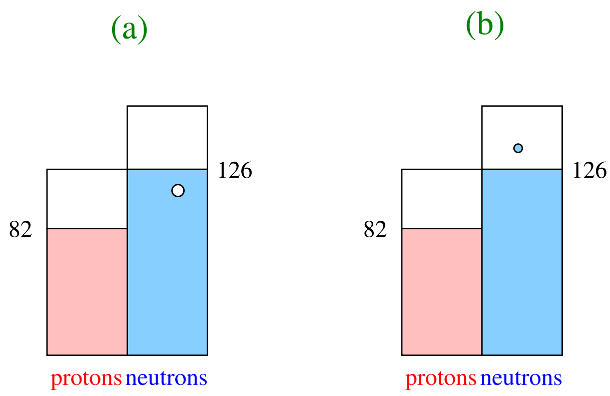

Figure 5.4: The shell structure of the one-neutron hole nucleus ^{207}Pb and the one-particle nucleus ^{209}Pb.

2- By looking at nuclei one particle or one hole away from a doubly magic nucleus. As an example look at the nuclei around 208Pb, as in Fig. 5.3.

In order to understand this figure we need to think a little about the shell structure, as sketched very schematically in Fig. 5.4: The one neutron-hole nucleus corresponds to taking away a single neutron from the 50-82 shell, and the one neutron particle state to adding a a neutron above the $N = 128$ shell closure. We can also understand more clearly why a closed shell nucleus has very few low-energy excited quantum states, since we would have to create a hole below the closed shell, and promote the nucleon in that shell to an open state above the closure. This requires an energy that equals the gap in the single-particle energy.

5.2 Collective models

Another, and actually older, way to look at nuclei is as a drop of "quantum fluid". This ignores the fact that a nucleus is made up of protons and neutrons, and explains the structure of nuclei in terms of a continuous system, just as we normally ignore the individual particles that make up a fluid.

5.2.1 Liquid drop model and mass formula

Now we have some basic information about the liquid drop model, let us try to reinterpret the mass formula in terms of this model; especially as those of a spherical drop of liquid.

As a prime example consider the Coulomb energy. The general energy associated with a charge distribution is

$$E_{\text{Coulomb}} = \tfrac{1}{2} \int \frac{\rho(r_1)\rho(r_2)}{4\pi\epsilon_0 r_{12}} d^3r_1 d^3r_2, \tag{5.6}$$

where the charge distribution is the smeared out charge of the protons,

$$4\pi \int_0^R \rho(r) r^2 dr = Ze \tag{5.7}$$

If we take the charge to be homogeneously distributed $\rho = Ze/(4/3\pi R^3)$, then

$$\begin{aligned}
E_{\text{Coulomb}} &= \frac{(2\pi)(4\pi)}{4\pi\epsilon_0}\rho^2 \int_0^R \int_{-1}^1 \frac{r_1^2\, dr_1\, d\cos\theta\, r_2^2\, dr_2}{\left(r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta\right)^{1/2}} \\
&= \frac{\pi\rho^2}{2\epsilon_0} \int_0^R \int_0^R \frac{(r_1 + r_2) - |r_1 - r_2|}{r_1 r_2} r_1^2 dr_1\, r_2^2 dr_2 \\
&= \frac{\pi\rho^2}{2\epsilon_0} \left[2 \int_0^R x^2 dx \int_0^R y dy - \int_0^R \int_0^R |r_1 - r_2| r_1 r_2 dr_1\, dr_2\right] \\
&= \frac{\pi\rho^2}{2\epsilon_0} \left(\frac{R^5}{3} - R^5 15\right) \\
&= \frac{\pi Z^2 9e^2}{(4\pi)^2 R^6 2\epsilon_0} \frac{4R^5}{15} = \frac{e^2}{4\pi\epsilon_0} \frac{3}{10} \frac{Z^2}{R}.
\end{aligned} \tag{5.8}$$

5.2.2 Equilibrium shape & deformation

Once we picture a nucleus as a fluid, we can ask question about its equilibrium shape. From experimental data we know that near closed shells nuclei are spherical, i.e., the equilibrium shape is a sphere. When both the proton and neutron number differ appreciably from the magic numbers, the ground state is often found to be axially deformed, either prolate (cigar like) or oblate (like a pancake).

A useful analysis to perform is to see what happens when we deform a nucleus slightly, turning it into an ellipsoid, with one axis slightly longer than the others, keeping a constant volume:

$$a = R(1+\epsilon), \qquad b = R(1+\epsilon)^{-1/2}. \tag{5.9}$$

The volume is $\frac{4}{3}\pi ab^2$, and is indeed constant. The surface area of an ellipsoid is more complicated, and we find

$$S = 2\pi \left[b^2 + ab\frac{\arcsin e}{e}\right], \tag{5.10}$$

where the eccentricity e is defined as

$$e = \left[1 - b^2/a^2\right]^{1/2}. \tag{5.11}$$

For small deformation ϵ we find a much simpler result,

$$S = 4\pi R^2 \left[1 + \frac{2}{5}\epsilon^2\right], \tag{5.12}$$

and the surface area thus increases for both elongations and contractions. Thus the surface energy increases by the same factor. There is one competing term, however, since the Coulomb energy also changes, the Coulomb energy goes down, since the particles are further apart,

$$E_{\text{Coulomb}} \to E_{\text{Coulomb}} \left(1 - \frac{\epsilon^2}{5}\right) \tag{5.13}$$

We thus find a change in energy of

$$DeltaE = \epsilon^2 \left[\frac{2}{5}\beta A^{2/3} - \frac{1}{5}\epsilon Z^2 A^{-1/3}\right] \tag{5.14}$$

The spherical shape is stable if $\Delta E > 0$.

Since it is found that the nuclear fluid is to very good approximation incompressible, the dynamical excitations are those where the shape of the nucleus fluctuates, keeping the volume constant, as well as those where the nucleus rotates without changing its intrinsic shape.

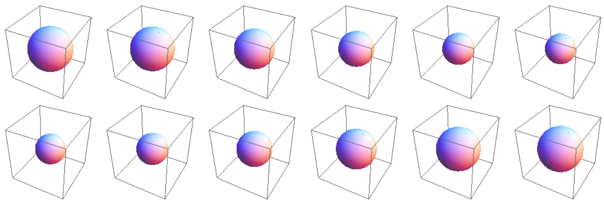

Figure 5.5: Monopole fluctuations of a liquid drop

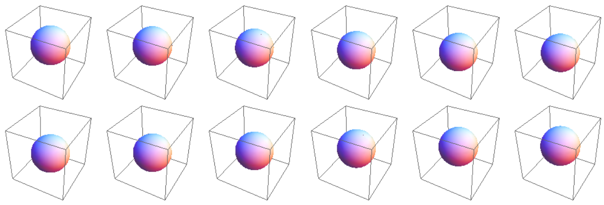

Figure 5.6: Dipole fluctuations of a liquid drop

5.2.3 Collective vibrations

Let us first look at collective vibrations, and for simplicity only at those of a spherical fluid drop. We can think of a large number of shapes; a complete set can be found by parametrising the surface as $r = \sum_{L,M} a_{LM} Y_{LM}(\theta, \phi)$, where Y_{LM} are the spherical harmonics and describe the multipolarity (angular momentum) of the surface. A few examples are shown in Figs. xxx, where we sketch the effects of monopole ($L = 0$), dipole ($L = 1$), quadrupole ($L = 2$) and octupole ($L = 3$) modes. Let us investigate these modes in turn, in the harmonic limit, where we look at small vibrations (small a_{LM}) only.

Monopole

The monopole mode, see Fig. 5.5, is the one where the size of the nuclear fluid oscillates, i.e., where the nucleus gets compressed. Experimentally one finds that the lowest excitation of this type, which in even-even nuclei carries the quantum number $J^\pi = 0^+$, occurs at an energy of roughly

$$E_0 \approx 80A^{-1/3}\text{MeV} \tag{5.15}$$

above the ground state. Compared to ordinary nuclear modes, which have energies of a few MeV, these are indeed high energy modes (15 MeV for $A = 216$), showing the incompressibility of the nuclear fluid.

Dipole

The dipole mode, Fig. 5.6, by itself is not very interesting: it corresponds to an overall translation of the centre of the nuclear fluid. One can, however, imagine a two-fluid model where a proton and neutron fluid oscillate against each other. This is a collective isovector ($I = 1$) mode. It has quantum numbers $J^\pi = 1^-$, occurs at an energy of roughly

$$E_0 \approx 77A^{-1/3}\text{MeV} \tag{5.16}$$

above the ground state, close to the monopole resonance. It shows that the neutron and proton fluids stick together quite strongly, and are hard to separate.

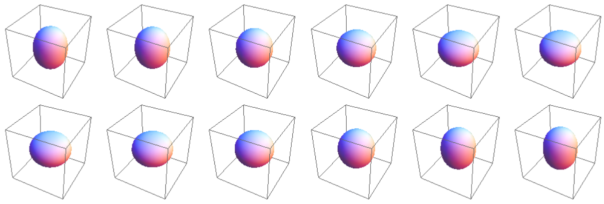

Figure 5.7: Quadrupole fluctuations of a liquid drop

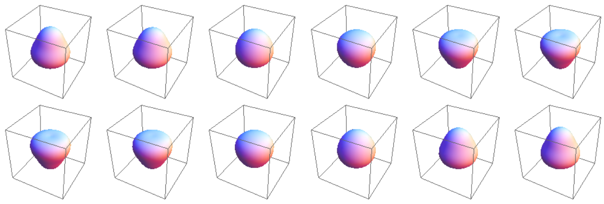

Figure 5.8: Octupole fluctuations of a liquid drop

Quadrupole

Quadrupole modes, see Fig. 5.7, are the dominant vibrational feature in almost all nuclei. The very special properties of the lower multipolarities mean that these are the first modes available for low-energy excitations in nuclei. In almost all even-even nuclei we find a low-lying state (at excitation energy of less than $1-2$MeV), which carries the quantum numbers $J^\pi = 2^+$, and near closed shells we can often distinguish the second harmonic states as well (three states with quantum numbers $J^\pi = 0^+, 2^+, 4^+$) .

Octupole

Octupole modes, with $J^\pi = 3^-$, see Fig. 5.8, can be seen in many nuclei. In nuclei where shell-structure makes quadrupole modes occur at very high energies, such as doubly magic nuclei, the octupole state is often the lowest excited state.

5.2.4 Collective rotations

Once we have created a nucleus with axial deformation, i.e., a nucleus with ellipsoidal shape, but still axial symmetry about one axis, we can rotate the fluid around one of the non-symmetry axes to generate excitations, see Fig. 5.9. We cannot do it around a symmetry axis, since the resulting state would just be the same quantum state as we started with, and therefore the energy cannot change. A rotated state around a non-symmetry axis is a different quantum state, and therefore we can overlay many of these

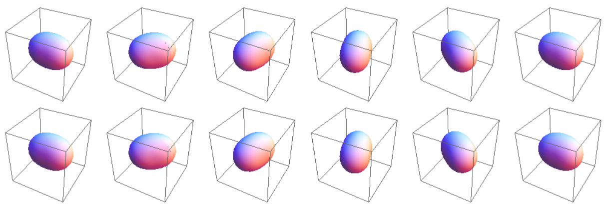

Figure 5.9: Collective rotation of an axially deformed liquid drop

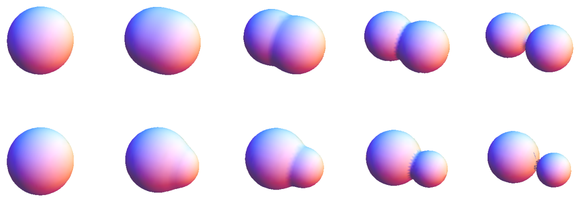

Figure 5.10: symmetric (upper row) or asymmetric fission (lower row)

states, especially with constant rotational velocity. This is almost like the rotation of a dumbbell, and we can predict the classical spectrum to be of the form

$$H = \frac{1}{2\mathcal{I}}\boldsymbol{J}^2, \tag{5.17}$$

where $\boldsymbol{J}$ is the classical angular momentum. We predict a quantum mechanical spectrum of the form

$$E_{\text{rot}}(J) = \frac{\hbar^2}{2\mathcal{I}}J(J+1), \tag{5.18}$$

where J is now the angular momentum quantum number. Naively we expect the spectrum to be more compressed (the moment of inertial is larger) the more elongated the nucleus becomes. It is known that certain structures in nuclei indeed describe well deformed nuclei, up to super and hyper deformed (axis ratio from 1 : 1.2 to 1 : 2).

5.3 Fission

Once we have started to look at the liquid drop model, we can try to ask the question what it predicts for fission, where one can use the liquid drop model to good effect. We are studying how a nuclear fluid drop separates into two smaller ones, either about the same size, or very different in size.

This process is indicated in Fig. 5.10. The liquid drop elongates, by performing either a quadrupole or octupole type vibration, but it persists until the nucleus falls apart into two pieces. Since the equilibrium shape must be stable against small fluctuations, we find that the energy must go up near the spherical form, as sketched in Fig. 5.11.

In that figure we sketch the energy - which is really the potential energy - for separation into two fragments, R is the fragment distance. As with any of such processes we can either consider classical fission decays for energy above the fission barrier, or quantum mechanical tunnelling for energies below the barrier. The method used in fission bombs is to use the former, by hitting a ^{235}U nucleus with a slow neutron a state with energy above the barrier is formed, which fissions fast. The fission products are unstable, and emit additional neutrons, which can give rise to a chain reaction.

The mass formula can be used to give an indication what is going on; Let us look at at the symmetric fusion of a nucleus. In that case the Q value is

$$Q = M(A,Z) - 2M(A/2, Z/2) \tag{5.19}$$

Please evaluate this for ^{236}U (92 protons). The mass formula fails in predicting the asymmetry of fission, the splitting process is much more likely to go into two unequal fragments.

Missing: Picture of asymmetric fusion.

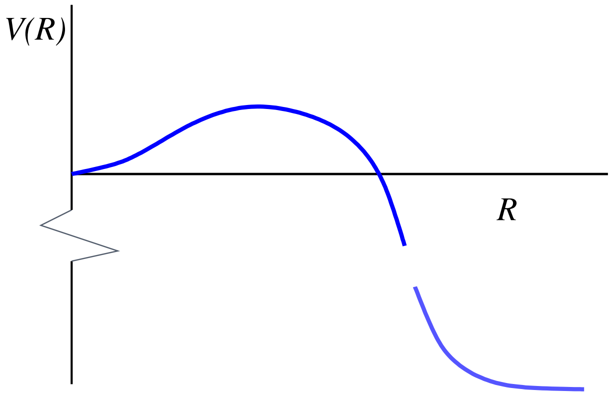

Figure 5.11: potential energy for fission

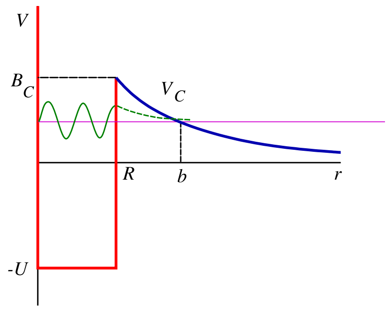

Figure 5.12: The potential energy for alpha decay

5.4 Barrier penetration

In order to understand quantum mechanical tunnelling in fission it makes sense to look at the simplest fission process: the emission of a He nucleus, so called α radiation. The picture is as in Fig. 5.12.

Suppose there exists an α particle inside a nucleus at an (unbound) energy > 0. Since it isn't bound, why doesn't it decay immediately? This must be tunnelling. In the sketch above we have once again shown the nuclear binding potential as a square well, but we have included the Coulomb tail,

$$V_{\text{Coulomb}}(r) = \frac{(Z-2)2e^2}{4\pi\epsilon_0 r}. \tag{5.20}$$

. The height of the barrier is exactly the coulomb potential at the boundary, which is the nuclear radius, $R_C = 1.2A^{1.3}$ fm, and thus $B_C = 2.4(Z-2)A^{-1/3}$. The decay probability across a barrier can be given by the simple integral expression $P = e^{-2\gamma}$, with

$$\begin{aligned}
\gamma &= \frac{(2\mu_\alpha)^{1/2}}{\hbar}\int_{R_C}^{b}[V(r)-E_\alpha]^{1/2}dr \\
&= \frac{(2\mu_\alpha)^{1/2}}{\hbar}\int_{R_C}^{b}\left[\frac{2(Z-2)e^2}{4\pi\epsilon_0 r}-E_\alpha\right]^{1/2}dr \\
&= \frac{2(Z-2)e^2}{2\pi\epsilon_0\hbar v}\left[\arccos(E_\alpha/B_C)-(E_\alpha/B_C)(1-E_\alpha/B_C)\right],
\end{aligned} \tag{5.21}$$

(here v is the velocity associated with E_α). In the limit that $B_C \gg E_\alpha$ we find

$$P = \exp\left[-\frac{2(Z-2)e^2}{2\epsilon_0\hbar v}\right]. \tag{5.22}$$

This shows how sensitive the probability is to Z and v!

Chapter 6

Some basic concepts of theoretical particle physics

We now come to the first hard part of the class. We'll try to learn what insights we can gain from the equation governing relativistic quantum mechanics.

6.1 The difference between relativistic and NR QM

One of the key points in particles physics is that special relativity plays a key rôle. As you all know, in ordinary quantum mechanics we ignore relativity. Of course people attempted to generate equations for relativistic theories soon after Schrödinger wrote down his equation. There are two such equations, one called the Klein-Gordon and the other one called the Dirac equation.

The structure of the ordinary Schrödinger equation of a free particle (no potential) suggests what to do. We can write this equation as

$$\hat{H}\psi = \frac{1}{2m}\boldsymbol{p}^2\psi = i\hbar\frac{\partial}{\partial t}\psi. \tag{6.1}$$

This is clearly a statement of the non-relativistic energy-momentum relation, $E = \frac{1}{2}mv^2$, since a time derivative on a plane wave brings down a factor energy. Remember, however, that $\boldsymbol{p}$ as an operator also contains derivatives,

$$\boldsymbol{p} = \frac{\hbar}{i}\boldsymbol{\nabla}. \tag{6.2}$$

A natural extension would to use the relativistic energy expression,

$$\hat{H}\psi = \sqrt{m^2c^4 + \boldsymbol{p}^2c^2}\,\psi = i\hbar\frac{\partial}{\partial t}\psi. \tag{6.3}$$

But this is a nonsensical equation, unless we specify how to take the square root of the operator. The first attempt to circumvent this problem, by Klein and Gordon, was to take the square of the equation,

$$\left(m^2c^4 + \boldsymbol{p}^2c^2\right)\psi = -\hbar^2\frac{\partial^2}{\partial t^2}\psi. \tag{6.4}$$

This is an excellent equation for spin-less particles or spin one particles (bosons), but not to describe fermions (half-integer spin), since there is no information about spin is in this equation. This needs careful consideration, since spin must be an intrinsic part of a relativistic equation!

Dirac realised that there was a way to define the square root of the operator. The trick he used was to define four matrices $\boldsymbol{\alpha}, \beta$ that each have the property that their square is one, and that they anticommute,

$$\begin{aligned} \alpha_i\alpha_i &= I, & \beta\beta &= I, \\ \alpha_i\beta + \beta\alpha_i &= 0, & \alpha_i\alpha_j + \alpha_j\alpha_i &= 0 \quad i \neq j. \end{aligned} \tag{6.5}$$

This then leads to an equation that is linear in the momenta – and very well behaved,

$$(\beta mc^2 + c\boldsymbol{\alpha}\cdot\boldsymbol{p})\Psi = i\hbar\frac{\partial}{\partial t}\Psi \tag{6.6}$$

Note that the minimum dimension for the matrices in which we can satisfy all conditions is 4, and thus Ψ is a four-vector! This is closely related to the fact that these particles have spin.

Let us investigate this equation a bit further. One of the possible forms of α_i and β is

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{6.7}$$

where σ_i are the two-by-two Pauli spin matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{6.8}$$

(These matrices satisfy some very interesting relations. For instance

$$\sigma_1\sigma_2 = i\sigma_3, \quad \sigma_2\sigma_1 = -i\sigma_3, \quad \sigma_2\sigma_3 = i\sigma_1, \tag{6.9}$$

etc. Furthermore $\sigma_i^2 = 1$.)

Once we know the matrices, we can try to study a plane-wave solution

$$\Psi(\boldsymbol{x}, t) = u(\boldsymbol{p})e^{i(\boldsymbol{p}\cdot\boldsymbol{x} - Et)/\hbar}. \tag{6.10}$$

(Note that the exponent is a "Lorentz scalar", it is independent of the Lorentz frame!).

If substitute this solution we find that $u(\boldsymbol{p})$ satisfies the eigenvalue equation

$$\begin{pmatrix} mc^2 & 0 & p_3c & p_1c - ip_2c \\ 0 & mc^2 & p_1c + ip_2c & -p_3c \\ p_3c & p_1c - ip_2c & -mc^2 & 0 \\ p_1c + ip_2c & -p_3c & 0 & -mc^2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = E \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix}. \tag{6.11}$$

The eigenvalue problem can be solved easily, and we find the eigenvalue equation

$$(m^2c^4 + p^2c^2 - E^2)^2 = 0 \tag{6.12}$$

which has the solutions $E = \pm\sqrt{m^2c^4 + p^2c^2}$. The eigenvectors for the positive eigenvalues are

$$\begin{pmatrix} 1 \\ 0 \\ p_3c/(E + mc^2) \\ (p_1c - ip_2c)/(E + mc^2) \end{pmatrix}, \text{ and } \begin{pmatrix} 0 \\ 1 \\ (p_1c + ip_2c)/(E + mc^2) \\ -p_3c/(E + mc^2) \end{pmatrix}, \tag{6.13}$$

with similar expressions for the two eigenvectors for the negative energy solutions. In the limit of small momentum the positive-energy eigenvectors become

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \text{ and } \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \tag{6.14}$$

and seem to denote a particle with spin up and down. We shall show that the other two solutions are related to the occurrence of anti-particles (positrons).

Just as photons are the best way to analyse (decompose) the electro-magnetic field, electrons and positrons are the natural way way to decompose the Dirac field that is the general solution of the Dirac equation. This analysis of a solution in terms of the particles it contains is called (incorrectly, for historical reasons) "second quantisation", and just means that there is a natural basis in which we can say there is a state at energy E, which is either full or empty. This could more correctly be referred to as the "occupation number representation" which should be familiar from condensed matter physics. This helps us to see how a particle can be described by these wave equations. There is a remaining problem, however!

6.2 Antiparticles

Both the Klein-Gordon and the Dirac equation have a really nasty property. Since the relativistic energy relation is quadratic, both equations have, for every positive energy solution, a negative energy solution. We don't really wish to see such things, do we? Energies are always positive and this is a real problem. The resolution is surprisingly simple, but also very profound – It requires us to look at the problem in a very different light.

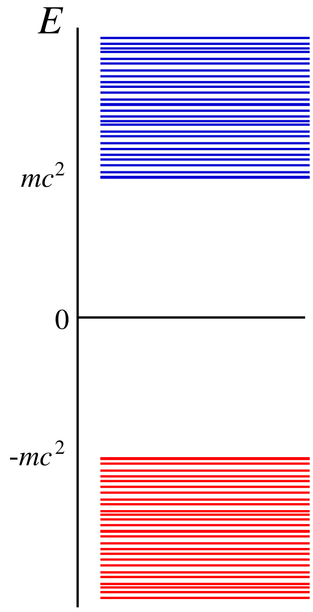

Figure 6.1: A schematic picture of the levels in the Dirac equation

In figure 6.1 we have sketched the solutions for the Dirac equation for a free particle. It has a positive energy spectrum starting at mc^2 (you cannot have a particle at lower energy), but also a negative energy spectrum below $-mc^2$. The interpretation of the positive energy states is natural – each state describes a particle moving at an energy above mc^2. Since we cannot have negative energy states, their interpretation must be very different. The solution is simple: We assume that in an *empty* vacuum all negative energy states are filled (the "Dirac sea"). Excitations relative to the vacuum can now be obtained by adding particles at positive energies, or creating *holes* at negative energies. Creating a hole takes energy, so the hole states appear at positive energies. They do have opposite charge to the particle states, and thus would correspond to positrons! This shows a great similarity to the behaviour of semiconductors, as you may well know. The situation is explained in figure 6.2.

Note that we have ignored the infinite charge of the vacuum (actually, we subtract it away assuming a constant positive background charge.) Removing infinities from calculations is a frequent occurrence in relativistic quantum theory (RQT). Many *unmeasurable* quantities become infinite, and we are only interested in the finite part remaining after removing the infinities. This process is part of what is called renormalisation, which is a systematic procedure to extract finite information from infinite answers!

6.3 QED: photon couples to e^+e^-

We know that electrons and positrons have charge and thus we need to include electrodynamics in the relativistic quantum theory of the electron. That is even more clear when we take into account that an

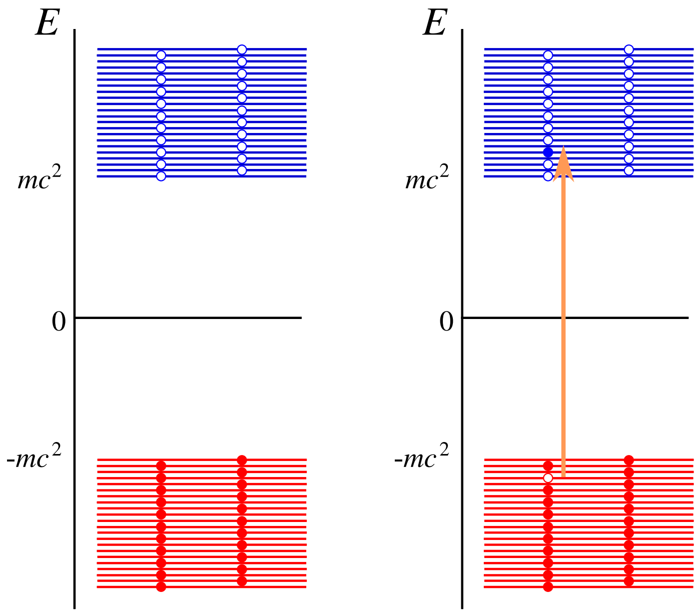

Figure 6.2: A schematic picture of the occupied and empty levels in the Dirac equation. The promotion of a particle to an empty level corresponds to the creation of a positron-electron pair, and takes an energy larger than $2mc^2$.

electron and positron can annihilate by emitting two photons (the well-known 511 keV lines),

$$e^+e^- \to \gamma\gamma. \tag{6.15}$$

Question: Why not one photon?

There is a natural way to describe this coupling, in a so-called Lagrangian approach, which I shall not discuss here. It teaches us that an electron can emit a photon, as indicated figure 6.3.

The diagrams in figure 6.3 are usually referred to as a Feynman diagrams, and the process depicted in (a) is usually called Bremsstrahlung, the one in (b) annihilation. With such a diagram comes a recipe for calculating it (called the Feynman rule). A key point is that energy and momentum are conserved in all reactions. Let us look at what happens when another nearby electron absorbs the photon, as in figure 6.4

Of course there are two possibilities: The left electron can emit the photon to the right one, or absorb one that is emitted by the right one. This is related to the time-ordering of interactions. One of the advantages of Feynman diagrams is that both these possibilities are described in one Feynman diagram. Thus the time in this diagram should only be interpreted in the sense of the external lines, what are the particles in and out. It is also very economical if we have more and more particles emitted and absorbed.

Since the emitted photon only lives for a short time, $\Delta t = \Delta x / c$, its energy cannot be determined exactly due to the uncertainty relation

$$\Delta E \Delta t \geq \frac{\hbar}{2}. \tag{6.16}$$

Thus even though the sum of the initial (four) momenta, $k_1 + k_2$ equals the sum of the final ones, $k_3 + k_4$, we find that the photon does *not* have to satisfy

$$q^2 = E_q^2 - \boldsymbol{q}^2 = 0. \tag{6.17}$$

Such a photon is called *virtual* or "off mass-shell", since it does not satisfy the mass-energy relations. This is what gives rise to the Coulomb force.

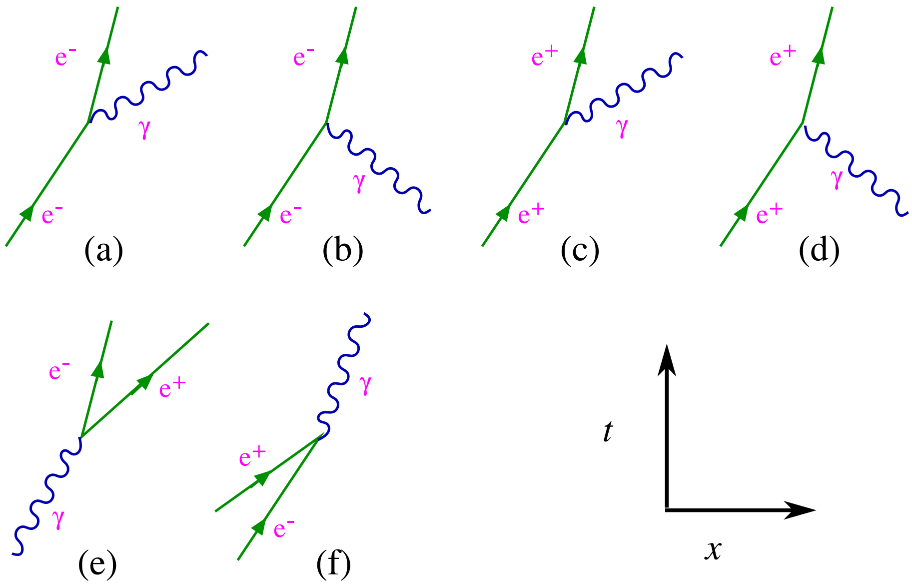

Figure 6.3: The Feynman diagrams for an electron and/or positrons interacting with a photon. Diagram (a) is emission of a photon by an electron, (b) absorption. (c) and (d) are the same diagrams for positrons, and (e) is pair creation, whereas (f) is annihilation.

6.4 Fluctuations of the vacuum

The great problem is in understanding the meaning of virtual particles. Suppose we are studying the vacuum state in QED. We wish to describe this vacuum in terms of the states of no positrons, electrons and photons (the naive vacuum). Since these particles interact we have short-lived states where e^+e^- pairs, and photons, and .... appear for a short while and disappear again. This is also true for real particles: a real electron is a "bare" electron surrounded by a cloud of virtual photons, e^+e^- pairs, etc. A photon can be an e^+e^- pair part of the time, and more of such anomalies.

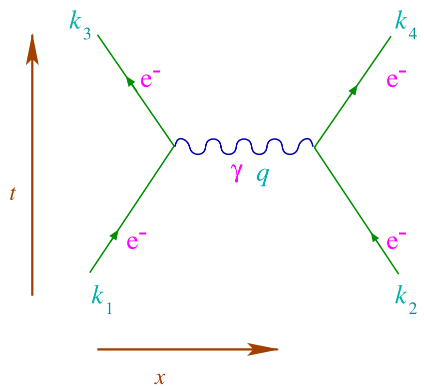

Figure 6.4: One of the Feynman diagrams for an electron-electron scattering.

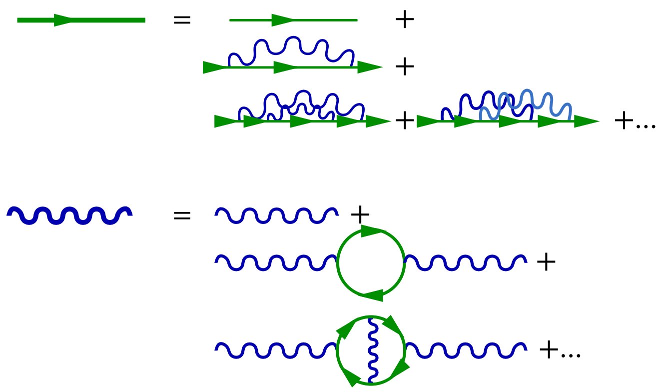

Figure 6.5: Some Feynman diagrams for "dressed propagators".

6.4.1 Feynman diagrams

As I have sketched above, Feynman diagrams can be used to describe what is happening in these processes. These describe the matrix elements, and the actual transition probability is proportional to the square of this matrix elements. One can show that each electron-photon coupling vertex is proportional to e, and thus in the square each vertex gives a factor e^2. Actually by drawing time in the vertical direction and space in the horizontal (schematically, of course), we see that the the two possible couplings of the photon to matter – Bremsstrahlung and pair creation are one and the same process. Still it helps to distinguish. Note that at each vertex charge is conserved as well as momentum!

This can actually be combined into a dimensionless quantity

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137}. \tag{6.18}$$

We should expand in α rather than e^2 since expansion parameters, being "unphysical" can not have dimensions. In other words in order to carry through this mathematical concept the natural scale of a diagram is set by the power of α it carries. Due to the smallness of α we normally consider only the diagrams with as few vertices as possible. Let me list the two diagrams for electron-positron scattering, both proportional to α^2, as given in figure 6.6.

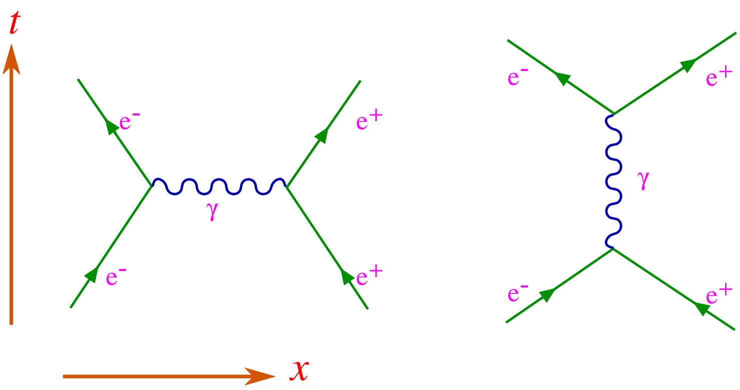

Figure 6.6: The two Feynman diagrams for an electron-positron scattering.

Question: Why is there only one such diagram for e^-e^- scattering? Answer: Charge conservation.

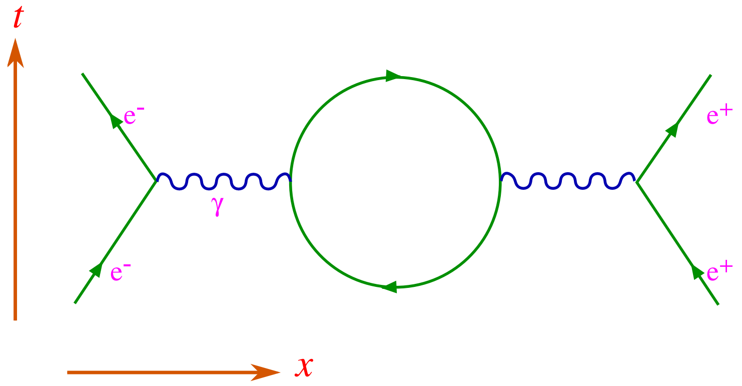

Figure 6.7: A higher order diagram for electron-positron scattering.

We can also construct higher order diagrams, as in figure 6.7.
We can also calculate the scattering of light by light, which only comes in at α^4, see figure 6.8.

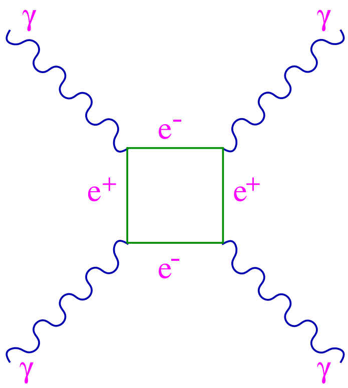

Figure 6.8: The lowest diagram for photon-photon scattering.

The sum of all diagrams contributing to a given process is called the perturbation series.

6.5 Infinities and renormalisation

One of the key features missing in the discussion above is the fact that all the pictures I have drawn are infinite – somewhat of a severe blow. The key point is to understand that this is not a problem, but has to do with a misinterpretation of the series.

When we introduce α and e in our theory these we use the measured value of the charge of an electron – which is a solution to the full theory, not to the artificial problem with all vacuum fluctuations turned of. What it means is that we should try to express all our answers in physically sensible (measurable) quantities. Renormalisation is the mathematical procedure that does this. A theory (such as QED) is called renormalisable if we can make all expressions finite by re-expressing them in a finite number of physical parameters.

6.6 The predictive power of QED

It is hard to say that a theory has predictive power without comparing it to experiment, so let me highlight a few successes of QED.

One of those is the so-called g factor of the electron, related to the ratio of the spin and orbital contributions to the magnetic moment. Relativistic theory (i.e., the Dirac equation) shows that $g = 2$. The measured value differs from 2 by a little bit, a fact well accounted for in QED.

$$\begin{array}{ll} \text{experiment} & g/2 = 1.00115965241(20) \\ \text{Theory} & g/2 = 1.00115965238(26) \end{array} \tag{6.19}$$

Some of the errors in the theory are related to our knowledge of constants such as $\hbar$, and require better input. It is also clear that at some scale QCD (the theory of strong interactions) will start playing a rôle. We are approaching that limit.

6.7 Problems

Example 6.1:

Discuss the number of different time-orderings of electron-positron scattering in lowest order in α.

Chapter 7

The fundamental forces

The fundamental forces are normally divided in four groups, of the four so-called "fundamental" forces. These are often naturally classified with respect to a dimensionless measure of their strength. To set these dimensions we use $\hbar$, c and the mass of the proton, m_p. The natural classification is then given in table 7.1. Another important property is their range: the distance to which the interaction can be felt, and the type of quantity they couple to. Let me look a little closer at each of these in turn.

Table 7.1: A summary of the four fundamental forces

Force	Range	Strength	Acts on
Gravity	∞	$G_N \approx 6\,10^{-39}$	All particles (mass and energy)
Weak Force	$< 10^{-18}$m	$G_F \approx 1\,10^{-5}$	Leptons, Hadrons
Electromagnetism	∞	$\alpha \approx 1/137$	All charged particles
Strong Force	$\approx 10^{-15}$m	$g^2 \approx 1$	Hadrons

In order to set the scale we need to express everything in a natural set of units. Three scales are provided by $\hbar$ and c and e – actually one usually works in units where these two quantities are 1 in high energy physics. For the scale of mass we use the mass of the proton. In summary (for $e = 1$ we use electron volt as natural unit of energy)

$$\hbar = 6.58 \times 10^{-22} \text{ MeV s} \quad (7.1)$$

$$\hbar c = 1.97 \times 10^{-13} \text{ MeV m} \quad (7.2)$$

$$m_\text{p} = 938 \text{ MeV}/c^2 \quad (7.3)$$

7.1 Gravity

The theory of gravity can be looked at in two ways: The old fashioned Newtonian gravity, where the potential is proportional to the rest mass of the particles,

$$V = \frac{G_N m_1 m_2}{r}. \quad (7.4)$$

We find that $G_N m_p^2/\hbar c$ is dimensionless, and takes on the value

$$G_N m_p^2/\hbar c = 5.9046486 \times 10^{-39}. \quad (7.5)$$

There are two more levels to look at gravity. One of those is Einstein's theory of gravity, which in the low-energy small-mass limit reduces to Newton's theory. This is still a *classical* theory, of a classical gravitational field.

The quantum theory, where we re-express the field in their quanta has proven to be a very tough stumbling block – When one tries to generalise the approach taken for QED, every expression is infinite, and one needs to define an infinite number of different infinite constants. This is not deemed to be acceptable – i.e., it doesn't define a theory. Such a model is called unrenormalisable. We may return to the problem of quantum gravity later, time permitting.

7.2 Electromagnetism

Electro-magnetism, i.e., QED, has been discussed in some detail in the previous chapter. Look there for a discussion. The coupling constant for the theory is

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}. \tag{7.6}$$

7.3 Weak Force

This manifests itself through nuclear β decay,

$$n \to p + e^- + \overline{\nu_e}. \tag{7.7}$$

The standard coupling for this theory is called the Fermi coupling, G_F, after its discoverer. After the theory was introduced it was discovered that there were physical particles that mediate the weak force, the $W^\pm$ and the Z^0 bosons. These are very heavy particles (their mass is about 80 times the proton mass!), which is why they have such a small range – fluctuations where I need to create that much mass are rare. The $W^\pm$ bosons are charged, and the Z^0 boson is neutral. The typical β decay referred to above is mediated by a W^- boson as can be seen in the Feynman diagram figure 7.1. The reason for this choice is that it conserves charge at each point (the charge of a proton and a W^- is zero, the charge of an electron and a neutrino is -1, the same as that of a W^-).

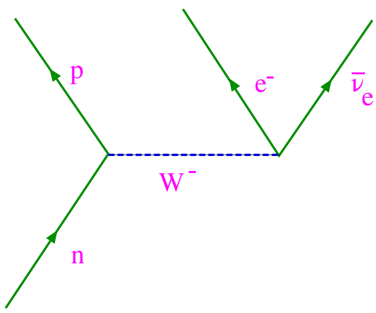

Figure 7.1: The Feynman diagram for the weak decay of a neutron.

7.4 Strong Force

The strong force is what keeps nuclei together. It is described by a theory called QCD, which described the forces between fermions called quarks that make up the hadrons. These forces are mediated by spin-1 bosons called gluons. Notice that this is a case where a series in powers of the coupling constant does not make a lot of sense, since higher powers have about the same value as lower powers. Such a theory is called non-perturbative.

Chapter 8

Symmetries and particle physics

Symmetries in physics provide a great fascination to us – one of the hang-ups of mankind. We can recognise a symmetry easily, and they provide a great tool to classify shapes and patterns. There is an important area of mathematics called group theory, where one studies the transformations under which an object is symmetric. In order to make this statement seem less abstract, let me look at a simple example, a regular hexagon in a plane. As can be seen in figure Fig. 8.5, this object is symmetric (i.e., we can't distinguish the new from the old object) under rotations around centre over angles of a multiple of 60°, and under reflection in any of the six axes sketched in the second part of the figure.

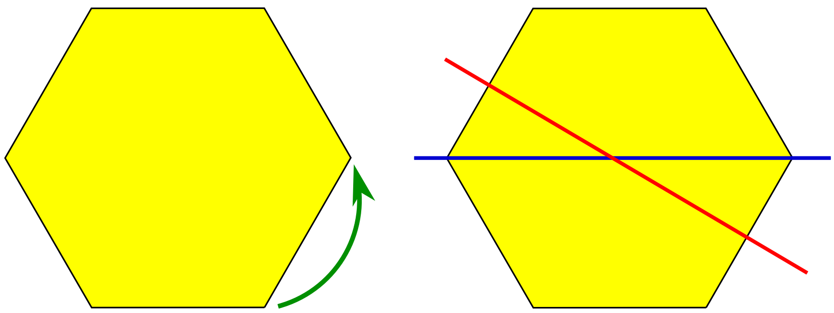

Figure 8.1: The symmetries of a hexagon

8.1 Importance of symmetries: Noether's theorem

There are important physical consequences of symmetries in physics, especially if the dynamics of a system is invariant under a symmetry transformation.

There is a theorem, due to Emily Noether, one of the most important (female) mathematicians of this century, that states that for any *continuous* symmetry there is a conserved quantity.

So what is a continuous symmetry? Think about something like spherical symmetry – a sphere is invariant under any rotation about its centre, no matter what the rotation angle. The continuity of choice of parameter in a transformation is what makes the set of transformations continuous. Another way of saying the same thing is that the transformation can be arbitrarily close to the unit transformation, i.e., it can do almost nothing at all.

8.2 Lorenz and Poincaré invariance

One of the most common continuous symmetries of a relativistic theory is Lorentz invariance, i.e., the dynamics is the same in any Lorentz frame. The group of Lorentz transformations can be decomposed into two parts: Pictures!

- Boosts, where we go from one Lorentz frame to another, i.e., we change the velocity.
- Rotations, where we change the orientation of the coordinate frame.

There is a slightly larger group of symmetries, called the Poincaré group, obtained when we add translations to the set of symmetries – clearly the dynamics doesn't care where we put the orbit of space.

The set of conserved quantities associated with this group is large. Translational and boost invariance implies conservation of four momentum, and rotational invariance implies conservation of angular momentum.

8.3 Internal and space-time symmetries

Above I have mentioned angular momentum, the vector product of position and momentum. This is defined in terms of properties of space (or to be more generous, of space-time). But we know that many particles carry the spin of the particle to form the total angular momentum,

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}. \tag{8.1}$$

The invariance of the dynamics is such that $\boldsymbol{J}$ is the conserved quantity, which means that we should not just rotate in ordinary space, but in the abstract "intrinsic space" where $\boldsymbol{S}$ is defined. This is something that will occur several times again, where a symmetry has a combination of a space-time and intrinsic part.

8.4 Discrete Symmetries

Let us first look at the key discrete symmetries – parity P (space inversion) charge conjugation C and time-reversal T.

8.4.1 Parity P

Parity is the transformation where we reflect each point in the origin, $\boldsymbol{x} \to -\boldsymbol{x}$. This transformation should be familiar to you. Let us think of the one dimensional harmonic oscillator, with Hamiltonian

$$-\frac{\hbar^2}{2m}\frac{d}{dx^2} + \tfrac{1}{2}m\omega^2 x^2. \tag{8.2}$$

The Hamiltonian does not change under the substitution $x \to -x$. The well-known eigenstates to this problem are either even or odd under this transformation, see Fig. 8.2, and thus have either even or odd parity,

$$P\psi(x,t) = \psi(-x,t) = \pm\psi(x,t), \tag{8.3}$$

where P is the transformation that take $x \to -x$. For

$$P\psi(x,t) = \psi(x,t) \tag{8.4}$$

we say that the state has even parity, for the minus sign we speak about negative parity. These are the only two allowed eigenvalues, as can be seen from looking at the probability density $|\psi(x,y)|^2$. Since this must be *invariant*, we find that

$$|\psi(x,y)|^2 = |P\psi(x,y)|^2 \tag{8.5}$$

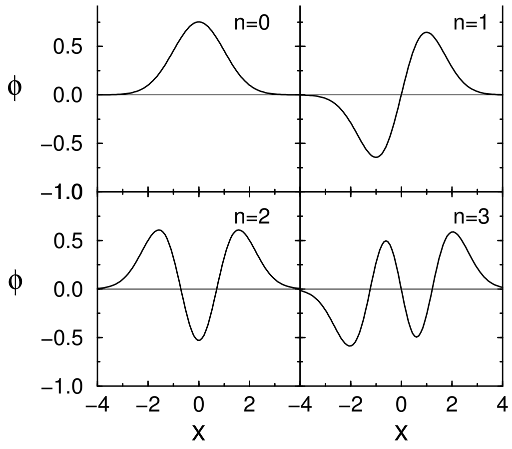

Figure 8.2: The first four harmonic oscillator wave functions

which shows that the only *real* eigenvalues for P are ± 1. One can show that there is a relation between parity and the orbital angular momentum quantum number L, $\pi = (-1)^L$, which relates two space-time symmetries.

It is found, however, that parity also has an intrinsic part, which is associated with each type of particle. A photon (γ) has negative parity. This can be understood from the following classical analogy. When we look at Maxwell's equation for the electric field,

$$\boldsymbol{\nabla} \cdot \boldsymbol{E}(x,t) = \frac{1}{\epsilon_0}\rho(x,t), \tag{8.6}$$

we find that upon reversal of the coordinates this equation becomes

$$-\boldsymbol{\nabla} \cdot \boldsymbol{E}(-x,t) = \frac{1}{\epsilon_0}\rho(-x,t). \tag{8.7}$$

The additional minus sign, which originates in the change of sign of ∇ is what gives the electric field and thus the photon its negative intrinsic parity.

We shall also wish to understand the parity of particles and antiparticles. For fermions (electrons, protons, ...) we have the interesting relation $P_f P_{\bar{f}} = -1$, which will come in handy later!

8.4.2 Charge conjugation C

The name of this symmetry is somewhat of a misnomer. Originally it stems from QED, where it was found that a set of interacting electrons behaves exactly the same way as a similar set of positrons. So if we change the sign of all charges the dynamics is the same. Actually, the symmetry generalises a little bit, and in general refers to a transformation where we change all particles in their antiparticles.

Once again we find $C^2 = 1$, and the only possible eigenvalues of this symmetry are ± 1. An uncharged particle like the photon that is its own antiparticle, must be an eigenstate of the symmetry operation, and it is found that it has eigenvalue -1,

$$C\psi_\gamma = -\psi_\gamma. \tag{8.8}$$

(Here ψ_γ is the wave function of the photon.) This can be shown from Maxwell's equation (8.6) as before, since ρ changes sign under charge conjugation.

For a combination of a particle and an antiparticle, we find that $C_f C_{\bar{f}} = -1$ for fermions, and $+1$ for bosons.

8.4.3 Time reversal T

On a microscopic scale it is not very apparent whether time runs forward or backwards, the dynamics where we just change the sign of time is equally valid as the original one. This corresponds to flipping the sign of all momenta in a Feynman diagram, so that incoming particles become outgoing particles and vice-versa. This symmetry is slightly nastier, and acts on both space-time and intrinsic quantities such as spin in a complicated way. The space time part is found to be

$$T\psi(\mathbf{r}, t) = \psi^*(\mathbf{r}, -t). \tag{8.9}$$

Combined with its intrinsic part we find that it has eigenvalues $\pm i$ for fermions (electrons, etc.) and ± 1 for bosons (photons, etc.).

8.5 The CPT Theorem

A little thought shows that all three symmetries mentioned above appear very natural – but that is a theorist's argument. The real key test is experiment, not a theorist's nice ideas! In 1956 C.N. Yang and T.D. Lee analysed the experimental evidence for these symmetries. They realised there was good evidence of these symmetries in QED and QCD (the theory of strong interactions). There was no evidence that parity was a symmetry of the weak interactions – which was true, since it was shown soon thereafter that these symmetries are broken, in a beautiful experiment led by "Madame" C.S. Wu.

There is a fairly strong proof that only minimal physical assumptions (locality, causality) that the product of C, P and T is a good symmetry of any theory. Up to now experiment has not shown any breaking of this product. We would have to rethink a lot of basic physics if this symmetry is not present. I am reasonably confident that if breaking is ever found there will be ten models that can describe it within a month!

8.6 CP violation

The first experimental confirmation of symmetry breaking was found when studying the β^- decay of ^{60}Co,

$$^{60}\text{Co} \rightarrow {}^{60}\text{Ni} + e^- + \bar{\nu}_e. \tag{8.10}$$

This nucleus has a ground state with non-zero spin, which can be oriented in a magnetic field.

A magnetic field is a pseudo-vector, which means that under parity it goes over into itself $\mathbf{B} \rightarrow \mathbf{B}$. So does the spin of the nucleus, and we thus have established that under parity the situation under which the nucleus emits electrons should be invariant. But the direction in which they are emitted changes! Thus any asymmetry between the emission of electrons parallel and anti-parallel to the field implies parity breaking, as sketched in figure 8.3.

Actually one can shown that to high accuracy that the product of C and P is conserved, as can be seen in figure 8.4.

8.7 Continuous symmetries

8.7.1 Translations

8.7.2 Rotations

8.7.3 Further study of rotational symmetry

Rotational transformations on a wave function can be applied by performing the transformation

$$\exp[i(\theta_x \hat{J}_x + \theta_y \hat{J}_y + \theta_z \hat{J}_z)] \tag{8.11}$$

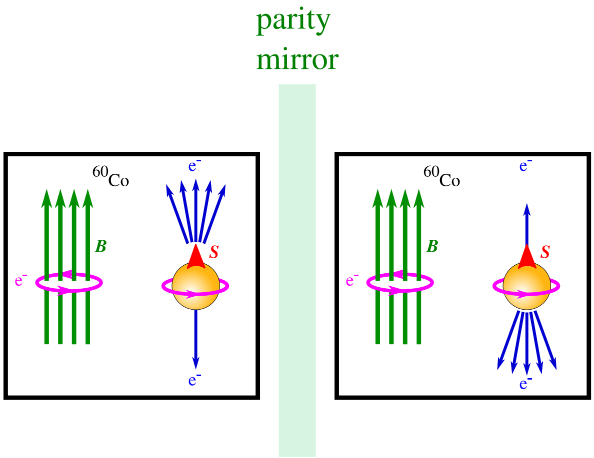

Figure 8.3: Parity breaking for the β decay of ^{60}Co

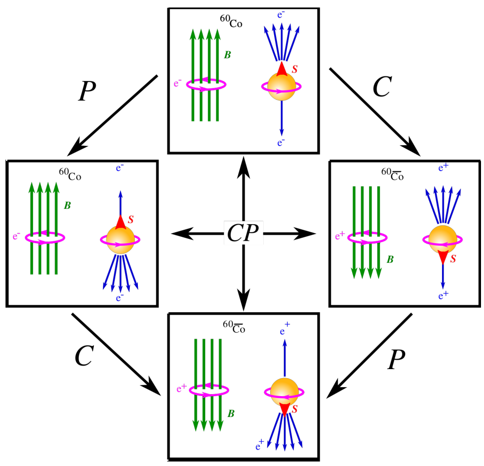

Figure 8.4: CP symmetry for the β decay of ^{60}Co

on a wave function. This is slightly simpler for a particle without spin, since we shall only have to consider the orbital angular momentum,

$$\hat{\boldsymbol{L}} = \hat{\boldsymbol{p}} \times \boldsymbol{r} = i\hbar\boldsymbol{r} \times \boldsymbol{\nabla}. \tag{8.12}$$

Notice that this is still very complicated, exponentials of operators are not easy to deal with. One of the lessons we learn from applying this operator to many different states, is that if a state has good angular momentum J, the rotation can transform it into another state of angular momentum J, but it will never change the angular momentum. This is most easily seen by labelling the states by J, M:

$$\left[\hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2\right]\phi_{JM} = \hbar^2 J(J+1)\phi_{JM} \tag{8.13}$$

$$\hat{J}_z^2\phi_{JM} = M\phi_{JM} \tag{8.14}$$

The quantum number M can take the values $-J, -J+1, \ldots, J-1, J$, so that we typically have $2J+1$ components for each J. The effect of the exponential transformation on a linear combination of states of identical J is to perform a linear transformation between these components. I shall show in a minute that such transformation can be implemented by unitary matrices. The transformations that implement these transformations are said to correspond to an irreducible representation of the rotation group (often denoted by SO(3)).

Let us look at the simplest example, for spin 1/2. We have two states, one with spin up and one with spin down, $\psi_\pm$. If the initial state is $\psi = \alpha_+\psi_+ + \alpha_-\psi_-$, the effect of a rotation can only be to turn this into $\psi' = \alpha'_+\psi_+ + \alpha'_-\psi_-$. Since the transformation is linear (if I rotate the sum of two objects, I might as well rotate both of them) we find

$$\begin{pmatrix} \alpha'_+ \\ \alpha'_- \end{pmatrix} = \begin{pmatrix} U_{++} & U_{-+} \\ U_{+-} & U_{--} \end{pmatrix} \begin{pmatrix} \alpha'_+ \\ \alpha'_- \end{pmatrix} \tag{8.15}$$

Since the transformation can not change the length of the vector, we must have $\int |\psi'|^2 = 1$. Assuming $\int |\psi_\pm|^2 = 1$, $\int \psi_+^*\psi_- = 0$ we find

$$U^\dagger U = 1 \tag{8.16}$$

with

$$U^\dagger = \begin{pmatrix} U^*_{++} & U^*_{+-} \\ U^*_{-+} & U^*_{--} \end{pmatrix} \tag{8.17}$$

the so-called hermitian conjugate.

We can write down matrices that in the space of $S = 1/2$ states behave the same as the angular momentum operators. These are *half* the well known Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{8.18}$$

and thus we find that

$$U(\boldsymbol{\theta}) = \exp[i(\theta_x\sigma_x/2 + \theta_y\sigma_y/2 + \theta_z\sigma_z/2)]. \tag{8.19}$$

I don't really want to discuss how to evaluate the exponent of a matrix, apart from one special case. Suppose we perform a 2π rotation around the z axis, $\boldsymbol{\theta} = (0, 0, 2\pi)$. We find

$$U(0, 0, 2\pi) = \exp[i\pi \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}]. \tag{8.20}$$

Since this matrix is diagonal, we just have to evaluate the exponents for each of the entries (this corresponds to using the Taylor series of the exponential),

$$\begin{aligned} U(0, 0, 2\pi) &= \begin{pmatrix} \exp[i\pi] & 0 \\ 0 & \exp[-i\pi] \end{pmatrix} \\ &= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned} \tag{8.21}$$

To our surprise this does not take me back to where I started from. Let me make a small demonstration to show what this means.........

Finally what happens if we combine states from two irreducible representations? Let me analyse this for two spin 1/2 states,

$$\begin{aligned} \psi &= (\alpha^1_+\psi^1_+ + \alpha^1_-\psi^1_-)(\alpha^2_+\psi^2_+ + \alpha^2_-\psi^2_-) \\ &= \alpha^1_+\alpha^2_+\psi^1_+\psi^2_+ + \alpha^1_+\alpha^2_-\psi^1_+\psi^2_- + \alpha^1_-\alpha^2_+\psi^1_-\psi^2_+\alpha^1_-\alpha^2_-\psi^1_-\psi^2_-. \end{aligned} \tag{8.22}$$

The first and the last product of ψ states have an angular momentum component ± 1 in the z direction, and must does at least have $J = 1$. The middle two combinations with both have $M = M_1 + M_2 = 0$ can be shown to be a combination of a $J = 1, M = 0$ and a $J = 0, M = 0$ state. Specifically,

$$\frac{1}{\sqrt{2}}\left[\psi^1_+\psi^2_- - \psi^1_-\psi^2_+\right] \tag{8.23}$$

transforms as a scalar, it goes over into itself. the way to see that is to use the fact that these states transform with the same U, and substitute these matrices. The result is proportional to where we started from. Notice that the triplet ($S = 1$) is symmetric under interchange of the two particles, whereas the singlet ($S = 0$) is antisymmetric. This relation between symmetry can be exhibited as in the diagrams Fig. **??**, where the horizontal direction denotes symmetry, and the vertical direction denotes antisymmetry. This technique works for all unitary groups.....

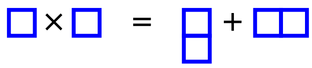

Figure 8.5: The Young tableau for the multiplication $1/2 \times 1/2 = 0 + 1$.

The coupling of angular momenta is normally performed through Clebsch-Gordan coefficients, as denoted by

$$\langle j_1 m_1 j_2 m_2 | J M \rangle \,. \tag{8.24}$$

We know that $M = m_1 + m_2$. Further analysis shows that J can take on all values $|j_1 - j_2|, |j_1 - j_2| + 1, |j_1 - j_2| + 2, j1 + j2$.

8.8 symmetries and selection rules

We shall often use the exact symmetries discussed up till now to determine what is and isn't allowed. Let us, for instance, look at

8.9 Representations of SU(3) and multiplication rules

A very important group is SU(3), since it is related to the colour carried by the quarks, the basic building blocks of QCD.

The transformations within SU(3) are all those amongst a vector consisting of three complex objects that conserve the length of the vector. These are all three-by-three unitary matrices, which act on the complex vector ψ by

$$\begin{aligned} \psi &\to U\psi \\ &= \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \end{aligned} \tag{8.25}$$

The complex conjugate vector can be shown to transform as

$$\psi^* \to \psi^* U^\dagger, \tag{8.26}$$

with the inverse of the matrix. Clearly the fundamental representation of the group, where the matrices representing the transformation are just the matrix transformations, the vectors have length 3. The representation is usually labelled by its number of basis elements as **3**. The one the transforms under the inverse matrices is usually denoted by $\bar{\mathbf{3}}$.

What happens if we combine two of these objects, ψ and χ^*? It is easy to see that the inner product of ψ and χ^* is scalar,

$$\chi^* \cdot \psi \to \chi^* U^\dagger U \psi = \chi^* \cdot \psi, \tag{8.27}$$

where we have used the unitary properties of the matrices the remaining 8 components can all be shown to transform amongst themselves, and we write

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}. \tag{8.28}$$

Of further interest is the product of three of these vectors,

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}. \tag{8.29}$$

8.10 broken symmetries

Of course one cannot propose a symmetry, discover that it is not realised in nature ("the symmetry is broken"), and expect that we learn something from that about the physics that is going on. But parity is broken, and we still find it a useful symmetry! That has to do with the manner in which it is broken, only weak interactions – the exchange of $W^{\pm}$ and Z bosons – break them. Any process mediated by strong, electromagnetic or (probably) gravitational forces conserves the symmetry. This is one example of a symmetry that is only mildly broken, i.e., where the conserved quantities are still recognisable, even though they are not exactly conserved.

In modern particle physics the way symmetries are broken teaches us a lot about the underlying physics, and it is one of the goals of grand-unified theories (GUTs) to try and understand this.

8.11 Gauge symmetries

One of the things I will not say much about, but which needs to be mentioned, is of a certain class of local symmetries (i.e., symmetries of the theory at each point in space and time) called gauge symmetries. This is a key idea in almost all modern particle physics theories, so much so that they are usually labelled by the local symmetry group. Local symmetries are not directly observable, and do not have immediate consequences. They allow for a mathematically consistent and simple formulation of the theories, and in the end predict the particle that are exchanged – the gauge particles, as summarised in table 8.1.

Table 8.1: The four fundamental forces and their gauge particles

Gravitation	graviton(?)
QED	photon
Weak	$W^{\pm}$, Z^0
Strong	gluons

Chapter 9

Symmetries of the theory of strong interactions

The first time people realised the key role of symmetries was in the plethora of particles discovered using the first accelerators. Many of those were composite particle (to be explained later) bound by the strong interaction.

9.1 The first symmetry: isospin

The first particles that show an interesting symmetry are actually the nucleon and the proton. Their masses are remarkably close,

$$M_p = 939.566\ \mathrm{MeV/c}^2 \quad M_n = 938.272\ \mathrm{MeV/c}^2. \tag{9.1}$$

If we assume that these masses are generated by the strong interaction there is more than a hint of symmetry here. Further indications come from the pions: they come in three charge states, and once again their masses are remarkably similar,

$$M_{\pi^+} = M_{\pi^-} = 139.567\ \mathrm{MeV/c}^2, \quad M_{\pi^0} = 134.974\ \mathrm{MeV/c}^2. \tag{9.2}$$

This symmetry is reinforced by the discovery that the interactions between nucleon (p and n) is independent of charge, they only depend on the nucleon character of these particles – the strong interactions see only one nucleon and one pion. Clearly a continuous transformation between the nucleons and between the pions is a symmetry. The symmetry that was proposed (by Wigner) is an internal symmetry like spin symmetry called isotopic spin or isospin. It is an abstract rotation in isotopic space, and leads to similar type of states with isotopic spin $I = 1/2, 1, 3/2, \ldots$. One can define the third component of isospin as

$$Q = e(I_3 + B), \tag{9.3}$$

where B is the baryon number ($B = 1$ for n, p, 0 for π). We thus find

	B	Q/e	I	I_3
n	1	0	1/2	−1/2
p	1	1	1/2	1/2
π^-	0	−1	1	−1
π^0	0	0	1	0
π^+	0	1	1	1

(9.4)

Notice that the energy levels of these particles are split by a magnetic force, as ordinary spins split under a magnetic force.

9.2 Strange particles

In 1947 the British physicists Rochester and Butler (from across the street) observed new particles in cosmic ray events. (Cosmic rays where the tool before accelerators existed – they are still used due to the unbelievably violent processes taking place in the cosmos. We just can't produce particles like that in the lab. (Un)fortunately the number of highly energetic particles is very low, and we won't see many events.) These particles came in two forms: a neutral one that decayed into a π^+ and a π^-, and a positively charge one that decayed into a μ^+ (heavy electron) and a photon, as sketched in figure 9.1.

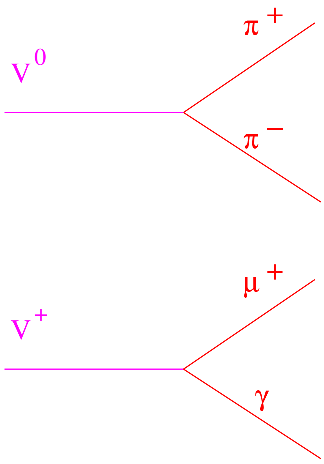

Figure 9.1: The decay of V particles

The big surprise about these particles was how long they lived. There are many decay time scales, but typically the decay times due to strong interactions are very fast, of the order of a femto second (10^{-15} s). The decay time of the K mesons was about 10^{-10} s, much more typical of a weak decay. Many similar particles have since been found, both of mesonic and baryonic type (like pions or like nucleons). These are collectively know as strange particles. Actually, using accelerators it was found that strange particles are typically formed in pairs, e.g.,

$$\pi^+ + p \rightarrow \underbrace{\Lambda^0}_{\text{baryon}} + \underbrace{K^0}\text{meson} \tag{9.5}$$

This mechanism was called associated production, and is highly suggestive of an additive conserved quantity, such as charge, called strangeness. If we assume that the Λ_0 has strangeness -1, and the K_0 $+1$, this balances

$$\begin{aligned} \pi^+ + p &\rightarrow \Lambda^0 + K^0 & (9.6)\\ 0 + 0 &= -1 + 1 & (9.7) \end{aligned}$$

The weak decay

$$\begin{aligned} \Lambda^0 &\rightarrow \pi^- + p & (9.8)\\ -1 &\neq 0 + 0, & (9.9) \end{aligned}$$

does not conserve strangeness (but it conserves baryon number). This process is indeed found to take much longer, about 10^{-10} s.

Actually it is found (by analysing many resonance particles) that we can accommodate this quantity in our definition of isospin,

$$Q = e(I_3 + \frac{B+S}{2}) \tag{9.10}$$

Clearly for $S = -1$ and $B = 1$ we get a particle with $I_3 = 0$. This allows us to identify the Λ^0 as an $I = 0, I_{3=0}$ particle, which agrees with the fact that there are no particles of different charge and a similar mass and strong interaction properties.

The kaons come in three charge states $K^\pm$, K^0 with masses $m_{K^\pm} = 494$ MeV, $m_{K^0} = 498$ MeV. In similarity with pions, which form an $I = 1$ multiplet, we would like to assume a $I = 1$ multiplet of K's as well. This is problematic since we have to assume $S = 1$ for all these particles: we cannot satisfy

$$Q = e(I_3 + \frac{1}{2}) \tag{9.11}$$

for isospin 1 particles. The other possibility $I = 3/2$ doesn't fit with only three particles. Further analysis shows that the the K^+ is the antiparticle of K^-, but K^0 is not its own antiparticle (which is true for the pions. So we need four particles, and the assignments are $S = 1, I = 1/2$ for K^0 and K^-, $S = -1, I = 1/2$ for K^+ and $\bar{K}^0$. Actually, we now realise that we can summarise all the information about K's and π's in one multiplet, suggestive of a (pretty badly broken!) symmetry.

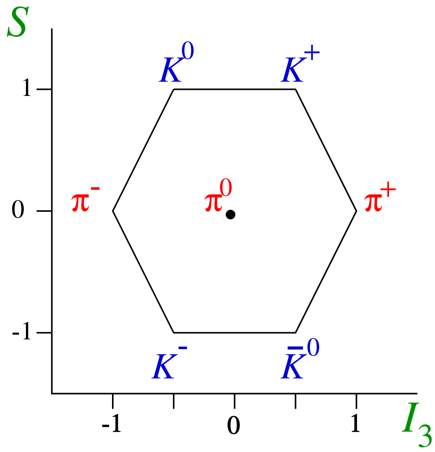

Figure 9.2: a possible arrangement for the states of the septet

However, it is hard to find a sensible symmetry that gives a 7-dimensional multiplet. It was argued by Gell-Mann and Ne'eman in 1961 that a natural extension of isospin symmetry would be an SU(3) symmetry. We have argued before that one of the simplest representations of SU(3) is 8 dimensional symmetry. A mathematical analysis shows that what is missing is a particle with $I = I_3 = S = 0$. Such a particle is known, and is called the η^0. The breaking of the symmetry can be seen from the following mass table:

$$\begin{aligned} m_{\pi^\pm} &= 139 \text{ MeV} \\ m_{\pi^0} &= 134 \text{ MeV} \\ m_{K^\pm} &= 494 \text{ MeV} \\ m_{\overset{(-)}{K^0}} &= 498 \text{ MeV} \\ m_{\eta^0} &= 549 \text{ MeV} \end{aligned} \tag{9.12}$$

The resulting multiplet is often represented like in figure 9.3.

In order to have the scheme make sense we need to show its predictive power. This was done by studying the nucleons and their excited states. Since nucleons have baryon number one, they are labelled with the "hyper-charge" Y,

$$Y = (B + S), \tag{9.13}$$

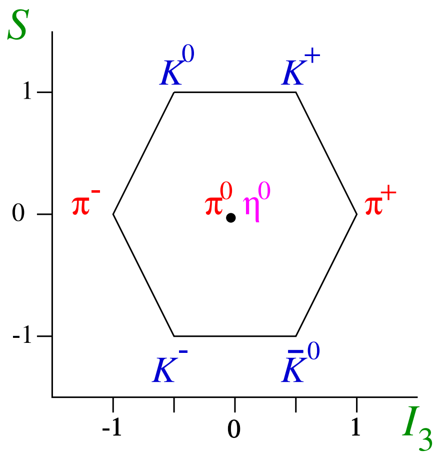

Figure 9.3: Octet of mesons

rather than S. The nucleons form an octet with the single-strangeness particles Λ and σ and the doubly-strange cascade particle Ξ, see figure 9.4.

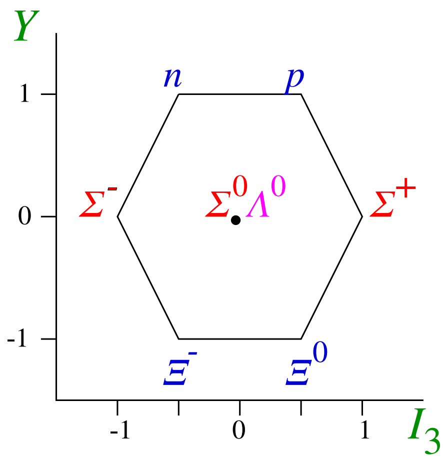

Figure 9.4: Octet of nucleons

The masses are

$$
\begin{aligned}
M_n &= 938 \text{ MeV} \\
M_p &= 939 \text{ MeV} \\
M_{\Lambda^0} &= 1115 \text{ MeV} \\
M_{\Sigma^+} &= 1189 \text{ MeV} \\
M_{\Sigma^0} &= 1193 \text{ MeV} \\
M_{\Sigma^-} &= 1197 \text{ MeV} \\
M_{\Xi^0} &= 1315 \text{ MeV} \\
M_{\Xi^-} &= 1321 \text{ MeV}
\end{aligned}
$$

All these particles were known before the idea of this symmetry. The first confirmation came when studying the excited states of the nucleon. Nine states were easily incorporated in a decuplet, and the tenth state (the Ω^-, with strangeness -3) was predicted. It was found soon afterwards at the predicted value of the mass.

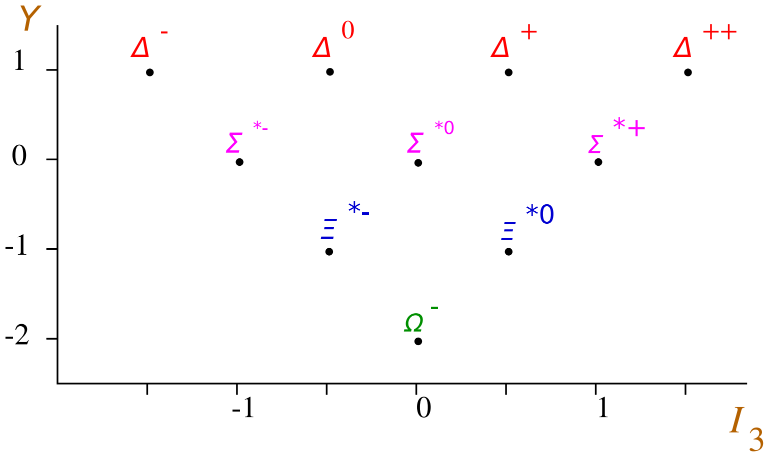

Figure 9.5: decuplet of excited nucleons

The masses are

$$
\begin{aligned}
M_\Delta &= 1232 \text{ MeV} \\
M_{\Sigma^*} &= 1385 \text{ MeV} \\
M_{\Xi^*} &= 1530 \text{ MeV} \\
M_\Omega &= 1672 \text{ MeV}
\end{aligned}
$$

(Notice almost that we can fit these masses as a linear function in Y, as can be seen in figure 9.6. This was of great help in finding the Ω.)

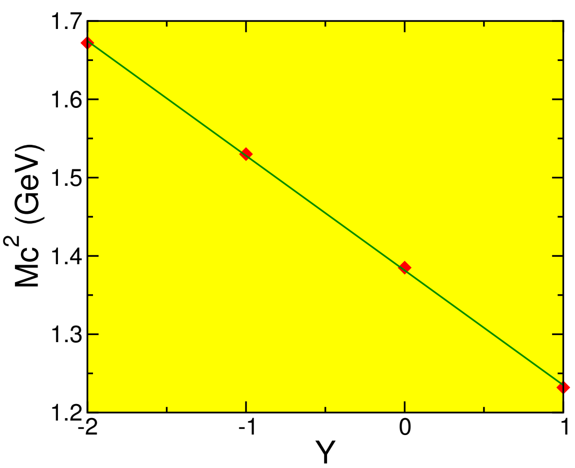

Figure 9.6: A linear fit to the mass of the decuplet

Table 9.1: The properties of the three quarks.

Quark	label	spin	Q/e	I	I_3	S	B
Up	u	$\frac{1}{2}$	$+\frac{2}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$	0	$\frac{1}{3}$
Down	d	$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	$\frac{1}{3}$
Strange	s	$\frac{1}{2}$	$-\frac{1}{3}$	0	0	-1	$\frac{1}{3}$

9.3 The quark model of strong interactions

Once the eightfold way (as the SU(3) symmetry was poetically referred to) was discovered, the race was on to explain it. As I have shown before the decaplet and two octets occur in the product

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}. \tag{9.14}$$

A very natural assumption is to introduce a new particle that comes in three "flavours" called up, down and strange (u, d and s, respectively), and assume that the baryons are made from three of such particles, and the mesons from a quark and anti-quark (remember, $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$.) Each of these quarks carries one third a unit of baryon number. The properties can now be tabulated, see table 9.2.

In the multiplet language I used before, we find that the quarks form a triangle, as given in Fig. 9.7.

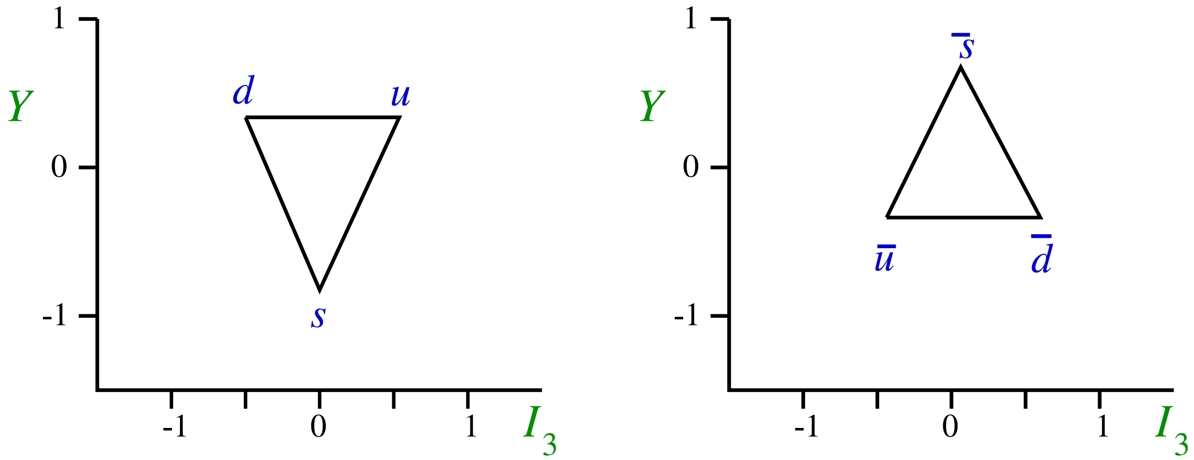

Figure 9.7: The multiplet structure of quarks and antiquarks

Once we have made this assignment, we can try to derive what combination corresponds to the assignments of the meson octet, figure 9.8. We just make all possible combinations of a quark and antiquark, apart from the scalar one $\eta' = u\bar{u} + d\bar{d} + c\bar{c}$ (why?).

A similar assignment can be made for the nucleon octet, and the nucleon decaplet, see e.g., see Fig. 9.9.

9.4 $SU(4), \ldots$

Once we have three flavours of quarks, we can ask the question whether more flavours exists. At the moment we know of three generations of quarks, corresponding to three generations (pairs). These give rise to SU(4), SU(5), SU(6) flavour symmetries. Since the quarks get heavier and heavier, the symmetries get more-and-more broken as we add flavours.

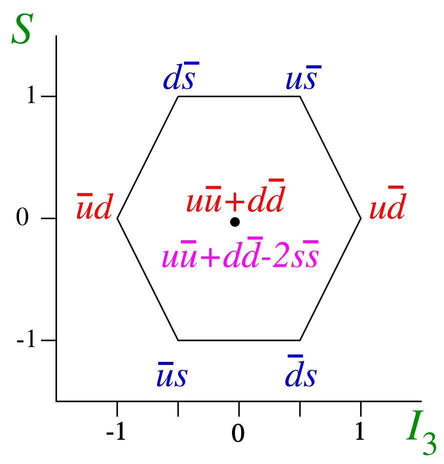

Figure 9.8: quark assignment of the meson octet

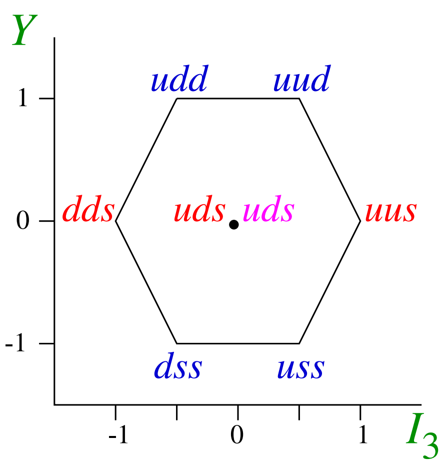

Figure 9.9: quark assignment of the nucleon octet

9.5 Colour symmetry

So why don't we see fractional charges in nature? This is an important point! In so-called deep inelastic scattering we see pips inside the nucleon – these have been identified as the quarks. We do not see any direct signature of individual quarks. Furthermore, if quarks are fermions, as they are spin 1/2 particles, what about antisymmetry of their wavefunction? Let us investigate the Δ^{++}, see Fig. 9.10, which consists of three u quarks with identical spin and flavour (isospin) and *symmetric* spatial wavefunction,

$$\psi_{\text{total}} = \psi_{\text{space}} \times \psi_{\text{spin}} \times \psi_{\text{flavour}}. \tag{9.15}$$

This would be symmetric under interchange, which is unacceptable. Actually there is a simple solution. We "just" assume that there is an additional quantity called colour, and take the colour wave function to be antisymmetric:

$$\psi_{\text{total}} = \psi_{\text{space}} \times \psi_{\text{spin}} \times \psi_{\text{flavour}} \times \psi_{\text{colour}} \tag{9.16}$$

We assume that quarks come in three colours. This naturally leads to yet another $SU(3)$ symmetry, which is actually related to the gauge symmetry of strong interactions, QCD. So we have shifted the question to: why can't we see coloured particles?

Table 9.2: The properties of the three quarks.

Quark	label	spin	Q/e	mass (GEV/c^2)
Down	d	$\frac{1}{2}$	$-\frac{1}{3}$	0.35
Up	u	$\frac{1}{2}$	$+\frac{2}{3}$	0.35
Strange	s	$\frac{1}{2}$	$-\frac{1}{3}$	0.5
Charm	c	$\frac{1}{2}$	$+\frac{2}{3}$	1.5
Bottom	b	$\frac{1}{2}$	$-\frac{1}{3}$	4.5
Top	t	$\frac{1}{2}$	$+\frac{2}{3}$	93

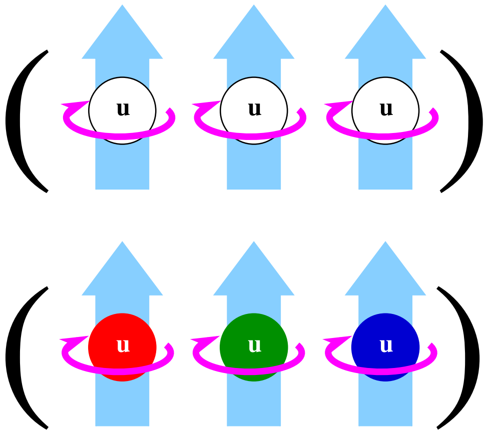

Figure 9.10: The Δ^{++} in the quark model.

This is a deep and very interesting problem. The only particles that have been seen are colour neutral ("white") ones. This leads to the assumption of confinement – We cannot liberate coloured particles at "low" energies and temperatures! The question whether they are free at higher energies is an interesting question, and is currently under experimental consideration.

9.6 The Feynman diagrams of QCD

There are two key features that distinguish QCD from QED:

1. Quarks interact more strongly the further they are apart, and more weakly as they are close by – asymptotic freedom.

2. Gluons interact with themselves

The first point can only be found through detailed mathematical analysis. It means that free quarks can't be seen, but at high energies quarks look more and more like free particles. The second statement make QCD so hard to solve. The gluon comes in 8 colour combinations (since it carries a colour and anti-colour index, minus the scalar combination). The relevant diagrams are sketches in Figure 9.11. Try to work out yourself how we satisfy colour charge conservation!

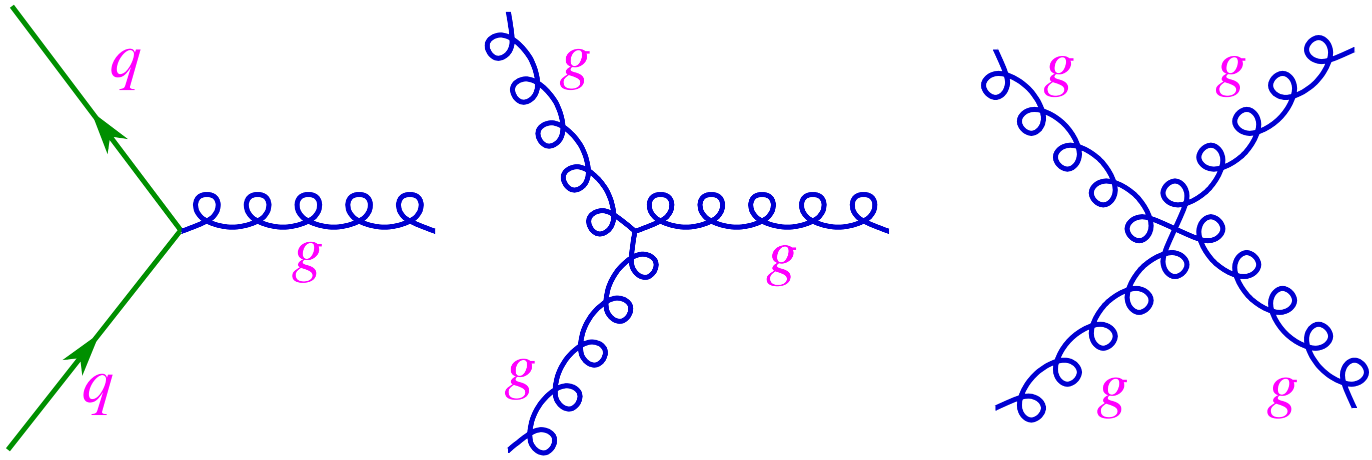

Figure 9.11: The basic building blocks for QCD Feynman diagrams

9.7 Jets and QCD

One way to see quarks is to use the fact that we can liberate quarks for a short time, at high energy scales. One such process is $e^+e^- \rightarrow q\bar{q}$, which use the fact that a photon can couple directly to $q\bar{q}$. The quarks don't live very long and decay by producing a "jet" a shower of particles that results from the decay of the quarks. These are all "hadrons", mesons and baryons, since they must couple through the strong interaction. By determining the energy in each if the two jets we can discover the energy of the initial quarks, and see whether QCD makes sense.

Chapter 10

Relativistic kinematics

One of the features of particle physics is the importance of special relativity. This occurs at a very fundamental level, since particle physics is all about creating and annihilating particles. This can only occur if we can convert mass to energy and vice-versa. Thus Einstein's idea of the equivalence between mass and energy plays an extremely fundamental rôle in this field of physics. In order for this to be possible we typically need processes that occur at velocities near the light velocity c, so that the kinematics (i.e., the description of momenta and energy) of these processes requires relativity. In this chapter we shall succinctly introduce the few necessary concepts – I hope that for most of you this is a review, but this chapter is intended to be self-contained and contains everything I shall need in relativistic kinematics.

10.1 Lorentz transformations of energy and momentum

As you may know, like we can combine position and time in one four-vector $x = (\boldsymbol{x}, ct)$, we can also combine energy and momentum in a single four-vector, $p = (\boldsymbol{p}, E/c)$. From the Lorentz transformation property of time and position, for a change of velocity along the x-axis from a coordinate system at rest to one that is moving with velocity $\boldsymbol{v} = (v_x, 0, 0)$ we have

$$x' = \gamma(v)(x - v/ct), \quad t' = \gamma(t - xvx/c^2), \tag{10.1}$$

we can derive that energy and momentum behave in the same way,

$$\begin{aligned} p'_x &= \gamma(v)(p_x - Ev/c^2) = mu_x\gamma(|u|), \\ E' &= \gamma(v)(E - vp_x) = \gamma(|u|)m_0c^2. \end{aligned} \tag{10.2}$$

To understand the context of these equations remember the definition of γ

$$\gamma(v) = 1/\sqrt{1-\beta^2}, \qquad \beta = \frac{v}{c}. \tag{10.3}$$

In Eq. (10.2) we have also re-expressed the momentum energy in terms of a velocity $\boldsymbol{u}$. This is measured relative to the rest system of a particle, the system where the three-momentum $\boldsymbol{p} = 0$.

Now all these exercises would be interesting mathematics but rather futile if there was no further information. We know however that the full four-momentum is conserved, i.e., if we have two particles coming into a collision and two coming out, the sum of four-momenta before and after is equal,

$$\begin{aligned} E_1^{\text{in}} + E_2^{\text{in}} &= E_1^{\text{out}} + E_2^{\text{out}}, \\ \boldsymbol{p}_1^{\text{in}} + \boldsymbol{p}_2^{\text{in}} &= \boldsymbol{p}_1^{\text{out}} + \boldsymbol{p}_2^{\text{out}}. \end{aligned} \tag{10.4}$$

10.2 Invariant mass

One of the key numbers we can extract from mass and momentum is the *invariant mass*, a number independent of the Lorentz frame we are in

$$W^2c^4 = (\sum_i E_i)^2 - (\sum_i \boldsymbol{p}_i)^2c^2. \tag{10.5}$$

This quantity takes it most transparent form in the centre-of-mass, where $\sum_i \boldsymbol{p}_i = 0$. In that case

$$W = E_{\mathrm{CM}}/c^2, \tag{10.6}$$

and is thus, apart from the factor $1/c^2$, nothing but the energy in the CM frame. For a single particle $W = m_0$, the rest mass.

Most considerations about processes in high energy physics are greatly simplified by concentrating on the invariant mass. This removes the Lorentz-frame dependence of writing four momenta. I

As an example we look at the collision of a proton and an antiproton at rest, where we produce two quanta of electromagnetic radiation (γ's), see fig. 10.1, where the anti proton has three-momentum $(p, 0, 0)$, and the proton is at rest.

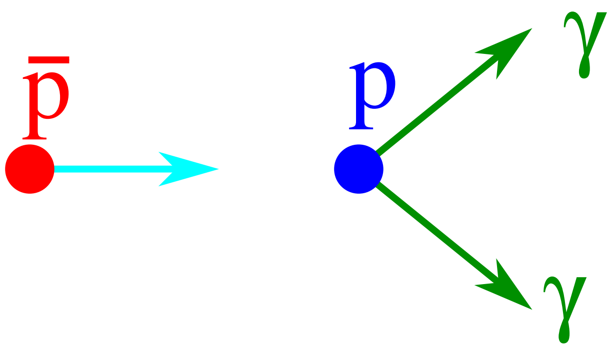

Figure 10.1: A sketch of a collision between a proton with velocity v and an antiproton at rest producing two *gamma* quanta.

The four-momenta are

$$\begin{aligned} p_{\mathrm{p}} &= (p_{\mathrm{lab}}, 0, 0, \sqrt{m_p^2c^4 + p_{\mathrm{lab}}^2c^2}) \\ p_{\bar{\mathrm{p}}} &= (0, 0, 0, m_pc^2). \end{aligned} \tag{10.7}$$

From this we find the invariant mass

$$W = \sqrt{2m_p^2 + 2m_p\sqrt{m_p^2 + p_{\mathrm{lab}}^2/c^2}} \tag{10.8}$$

If the initial momentum is much larger than m_p, more accurately

$$p_{\mathrm{lab}} \gg m_pc, \tag{10.9}$$

we find that

$$W \approx \sqrt{2m_pp_{\mathrm{lab}}/c}, \tag{10.10}$$

which energy needs to be shared between the two photons, in equal parts. We could also have chosen to work in the CM frame, where the calculations get a lot easier.

10.3 Transformations between CM and lab frame

Even though the use of the invariant mass simplifies calculations considerably, it clearly does not provide all necessary information. It does suggest however, that a natural frame to analyse reactions is the CM frame. Often we shall analyse a process in this frame, and use a Lorentz transformation to get information about processes in the laboratory frame. Since almost all processes involve the scattering (deflection) of one particle by another (or a number of others), this is natural example for such a procedure, see the sketch in Fig. 10.2. The same procedure can also be applied to the case of production of particles, such as the annihilation process discussed above.

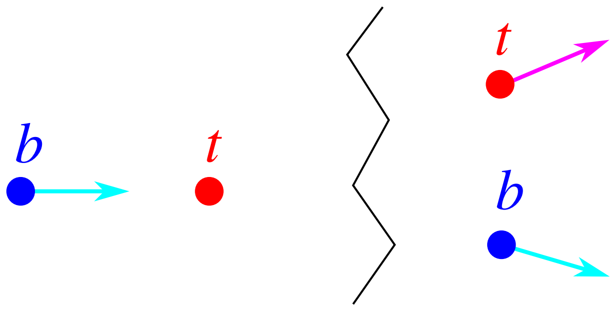

Figure 10.2: A sketch of a collision between two particles

Before the collision the beam particle moves with four-momentum

$$p_b = (p_{\mathrm{lab}}, 0, 0, \sqrt{m_b^2 c^4 + p_{\mathrm{lab}}^2 c^2}) \tag{10.11}$$

and the target particle m_t is at rest,

$$p_t = (0, 0, 0, m_t c^2). \tag{10.12}$$

We first need to determine the velocity v of the Lorentz transformation that bring is to the centre-of-mass frame. We use the Lorentz transformation rules for momenta to find that in a Lorentz frame moving with velocity v along the x-axis relative to the CM frame we have

$$\begin{aligned} p'_{bx} &= \gamma(v)(p_{\mathrm{lab}} - vE_{\mathrm{lab}}/c^2) \\ p'_{tx} &= -m_t v \gamma(v). \end{aligned} \tag{10.13}$$

Sine in the CM frame these numbers must be equal in size but opposite in sign, we find a linear equation for v, with solution

$$v = \frac{p_{\mathrm{lab}}}{m_t + E_{\mathrm{lab}}/c^2} \approx c\left(1 - \frac{m_t}{p_{\mathrm{lab}}}\right). \tag{10.14}$$

Now if we know the momentum of the beam particle in the CM frame after collision,

$$(p_f \cos\theta_{\mathrm{CM}}, p_f \sin\theta_{\mathrm{CM}}, 0, E'_f), \tag{10.15}$$

where θ_{CM} is the CM scattering angle we can use the inverse Lorentz transformation, with velocity $-v$, to try and find the lab momentum and scattering angle,

$$\begin{aligned} \gamma(v)(p_f \cos\theta_{\mathrm{CM}} + vE'_f/c^2) &= p_{f\mathrm{lab}} \cos\theta_{\mathrm{lab}} \\ p_f \sin\theta_{\mathrm{CM}} &= p_{f\mathrm{lab}} \sin\theta_{\mathrm{lab}}, \end{aligned} \tag{10.16}$$

from which we conclude

$$\tan\theta_{\mathrm{lab}} = \frac{1}{\gamma(v)} \frac{p_f \sin\theta_{\mathrm{CM}}}{p_f \cos\theta_{\mathrm{CM}} + vE'_f/c^2}. \tag{10.17}$$

Of course in experimental situations we shall often wish to transform from lab to CM frames, which can be done with equal ease.

To understand some of the practical consequences we need to look at the ultra-relativistic limit, where $p_{\rm lab} \gg m/c$. In that case $v \approx c$, and $\gamma(v) \approx (p_{\rm lab}/2m_t c^2)^{1/2}$. This leads to

$$\tan\theta_{\rm lab} \approx \sqrt{\frac{2m_t c^2}{p_{\rm lab}}} \frac{u\sin\theta_C}{u\cos\theta_C + c} \tag{10.18}$$

Here u is the velocity of the particle in the CM frame. This function is always strongly peaked in the forward direction unless $u \approx c$ and $\cos\theta_C \approx -1$.

10.4 Elastic-inelastic

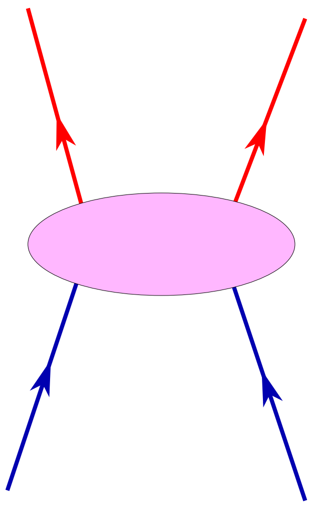

Figure 10.3: A sketch of a collision between two particles

We shall often be interested in cases where we transfer both energy and momentum from one particle to another, i.e., we have inelastic collisions where particles change their character – e.g., their rest-mass. If we have, as in Fig. 10.3, two particles with energy-momentum k_1 and p_q coming in, and two with k_2 and p_2 coming out, We know that since energy and momenta are conserved, that $k_1 + p_1 = k_2 + p_2$, which can be rearranged to give

$$p_2 = p_1 + q, \quad k_2 = k_1 - q. \tag{10.19}$$

and shows energy and momentum getting transferred. This picture will occur quite often!

10.5 Problems

Example 10.1:

Suppose a pion decays into a muon and a neutrino,

$$\pi^+ = \mu^+ + \nu_\mu. \tag{10.20}$$

Express the momentum of the muon and the neutrino in terms of the mass of pion and muon. Assume that the neutrino mass is zero, and that the pion is at rest. Calculate the momentum using $m_{\pi^+} = 139.6\ {\rm MeV}/c^2$, $m_\mu = 105.7\ {\rm MeV}/c^2$.

Example 10.2:

Calculate the lowest energy at which a $\Lambda(1115)$ can be produced in a collision of (negative) pions with protons at rest, throught the reaction $\pi^- + p \rightarrow K^0 + \Lambda$. $m_{\pi^-} = 139.6$ MeV/c^2, $m_p = 938.3$ MeV/c^2, $m_{K^0} = 497.7$ MeV/c^2. (Hint: the mass of the Λ is 1115 MeV/c^2.)

Example 10.3:

a) Find the maximum value for v such that the relativisitic energy can be expressed by

$$E \approx mc^2 + \frac{p^2}{2m}, \tag{10.21}$$

with an error of one procent.
b) find the minimum value of v and γ so that the relativisitic energy can be expressed by

$$E \approx pc, \tag{10.22}$$

again with an error of one percent.