Workshop 10, Week 10

Please follow the instructions of your supervisor regarding timing of these problems.

1. Laguerre's equation is

$$xy'' + (1 - x)y'(x) + \alpha y(x) = 0 \quad .$$

    Apply Frobenius method to this equation.
    (i) Find the indicial equation,
    (ii) and show that there is only one solution of generalised series form.
    (iii) Write the recurrence relation for the coefficients,
    (iv) and determine when the solution is a finite polynomial.
    (v) Determine the three simplest polynomials.

2. Verify that the following two functions are a solution of Laplace's equation ($\Delta f = 0$) in three dimensions:
    (i) $f_1 = \frac{1}{r}$;
    (ii) $f_2 = \frac{1}{2r} \ln \left[\frac{r+z}{r-z} \right]$.
    Hint: Write the equation in spherical coordinates. Use $z = r \cos \theta$.