2C1=P303=PH-2003 Workshop 11, Week 11

Please follow the instructions of your supervisor regarding timing of these problems.

1. Write the following differential equations in self-adjoint form. What is the orthogonality relation between the solutions?

   1. $(1-x^2)y''-2xy'+n(n+1)y=0$. (Legendre's equation, $-1<x<1$)
   2. $(1-x^2)y''-xy'+n^2y=0$. (Chebyshev's equation, $-1<x<1$)
   3. $xy''+(1-x)y'+ny=0$. (Laguerre's equation, $0<x<\infty$)
   4. $y''-2xy'+2ny=0$. (Hermite's equation, $-\infty<x<\infty$))

2. Consider a plate insulated laterally with radius C. The temperature $u(\rho,t)$ in the plate satisfies the differential equation (we assume that u is independent of ϕ)

   $$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial u}{\partial\rho}\right)-\frac{4}{\rho^2}u=\frac{1}{k}\frac{\partial u}{\partial t}$$

   (Heat is generated in the plate). Solve for u if $u(C)=0$ and the initial temperature is $f(\rho)$.

3. Determine A_j in the following Fourier-Bessel series

   1. $100=\sum_{j=1}^{\infty}A_jJ_0(\alpha_jx)\ \ \alpha_j=x_j/c, J_0(x_j)=0.$
   2. $x=\sum_{j=1}^{\infty}A_jJ_1(\alpha_jx)\ \ \alpha_j=x_j/5\ (c=5),\ J_1(x_j)=0.$