Workshop 12, Week 12

Please follow the instructions of your supervisor regarding timing of these problems.

1. 1. Write Laplace's equation in three dimensions if the function $u(r, \phi, \theta)$ is independent of θ.
   2. Write Laplace's equation in three dimensions if the function $u(r, \phi, \theta)$ is independent of ϕ and θ.

2. Show that

$$\int_{-1}^{1} f(x)P_n(x)dx = \frac{(-1)^n}{2^n n!}\int_{-1}^{1}(x^2-1)^n \frac{d^n}{dx^n}f(x)dx.$$

Hint: Use Rodrigue's formula

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n.$$

and partial integration.
Now show that if $f(x)$ is a polynomial of degree less than n,

$$\int_{-1}^{1} f(x)P_n(x)dx = 0.$$

3. A circular trampoline is 5 meters in diameter and is fixed to a frame. An acrobat jumps on the centre of the trampoline. By the time the canvas has returned to equilibrium position the canvas has an upward velocity of $(5-\rho)10$ m/s. Find the displacement $u(\rho, t)$.
Hint: Take the time that the canvas is horizontal as $t = 0$. Write the initial and boundary conditions. Solve the equation.