Workshop 2, Week 2

Please follow the instructions of your supervisor regarding timing of these problems.

1. Classify the following equations as elliptic, hyperbolic or parabolic.

   (i) The wave equation (1 space and 1 time dimension) $\nabla^2 u = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$
   (ii) The heat or diffusion equation, (1 space and 1 time dimension) $\nabla^2 u = \frac{1}{k}\frac{\partial u}{\partial t}$.
   (iii) Laplace's equation (2 space dimensions) $\nabla^2 u = 0$.
   (iv) Helmholtz's equation (2 space dimensions) $\nabla^2 u + \lambda u = 0$.
   (v) Poisson's equation (2 space dimensions) $\nabla^2 u = f(x, y)$.
   (vi) Time-independent Schrödinger equation (2 space dimensions) $\nabla^2 u - \alpha[E - V(x, y)]u = 0$.
   (vii) Klein-Gordon equation (1 space and 1 time dimension) $\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} + \lambda^2 u = 0$.

2. Classify the following differential equations

   (i) $$\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial y^2} + 2\frac{\partial^2 u}{\partial x \partial y} = 0.$$

   (ii) $$x\frac{\partial^2 u}{\partial x^2} + 2y\frac{\partial^2 u}{\partial y^2} + 2\frac{\partial^2 u}{\partial x \partial y} = 0.$$

   (iii) $$\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial y^2} + 5\frac{\partial^2 u}{\partial z^2} + 2\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial x \partial z} + 2\frac{\partial^2 u}{\partial y \partial z} + \frac{\partial u}{\partial x} + 3u = 0.$$

3. Solve $y' - xy = x$.

4. The Schrödinger equation for a constant potential is given by

   $$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0\psi = E\psi \quad .$$

   Give the general solution to this equation. Which solutions are physically acceptable? Interpret the solutions.

5. Solve

   $$\frac{d^2 y}{dx^2} - 4\frac{dy}{dx} + 4y = 0 \quad .$$