Workshop 6, Week 6

Please follow the instructions of your supervisor regarding timing of these problems.

1. Solve the equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

subject to the conditions

$$\begin{aligned} u(0,t) = u(l,t) = 0, &\qquad && t \geq 0, \\ u(x,0) = \alpha \sin\left(\frac{\pi x}{l}\right), &\qquad && 0 < x < l, \\ \frac{\partial u}{\partial t}(x,0) = 0, &\qquad && 0 < x < l. \end{aligned}$$

2. Find all solutions of

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} = u$$

which take the form

$$u(x,y) = \left(A\cos(\lambda x) + B\sin(\lambda x)\right) f(y).$$

(I.e., determine f and λ!)
Use this find the solution that satisfies

$$u(0,y) = u(\pi,y) = 0, \qquad u(x,1) = x.$$

3. The Schrödinger equation in one space and one time dimension is given by ($i^2 = -1$)

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}(x,t) + V(x)\psi(x,t) = \frac{\hbar}{i}\frac{\partial \psi}{\partial t}(x,t).$$

Use separation of variables to derive the time-independent Schrödinger equation. Solve for the time dependence. What determines the separation constants, and what is their dimension?