Workshop 7, Week 7

Please follow the instructions of your supervisor regarding timing of these problems.

1. The length of a guitar string is 65 cm. If the string is plucked 15 cm from the bridge, by raising it 3mm, find the displacement $u(x,t)$ by usin separaryion of variables

2. A quantum mechanical particle is confined inside a rectangular box of sides a, b, and c. The particle is described by a wave function ϕ, that satisfies Schrödinger's equation,

$$-\frac{\hbar^2}{2m}\nabla^2\phi = E\phi,$$

which has to vanish on the boundaries of the box. Investigate, using separation of variables, what the solutions looks like. Show that the lowest possible energy is

$$E = \frac{\pi^2\hbar^2}{2m}\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right).$$

3. Show that the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

admits solutions of the types D'Alambert's form, $f(x-ct)+g(x+ct)$, with f and g arbitrary. How can this be squared with a solution of separable form, say $A\sin(\frac{n\pi x}{L})\sin(\frac{n\pi ct}{L})$?