Workshop 8, Week 8

Please follow the instructions of your supervisor regarding timing of these problems.

1. (i) Show that the wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

admits solutions of D'Alambert's form, $f(x-ct)+g(x+ct)$, with f and g arbitrary.
(ii) How can this be squared with the existence of a solution of separable form, say $u_n(x,t) = A\sin(\frac{n\pi x}{L})\sin(\frac{n\pi ct}{L})$?

2. Determine the temperature $u(\rho,\phi)$ in a laterally insulated pie-point shaped region of radius c. The temperature satisfies the equation

$$\rho^2\frac{\partial^2 u}{\partial \rho^2} + \rho\frac{\partial u}{\partial \rho} + \frac{\partial^2 u}{\partial \phi^2} = 0$$

with boundary conditions

$$\begin{aligned} &u(\rho,0) = 0, \qquad 0 \le \rho \le c, \qquad u(\rho,\pi/6) = 0, \quad 0 \le \rho \le c,\\ &u(c,\phi) = \phi, \qquad 0 < \phi < \pi/6. \end{aligned}$$

Use separation of variables to find the solution.