QM I: Homework 1

Please hand in **before** Friday, 30 October 1998, 12 noon, in the departmental office.

1. Normalise the solutions of the time-independent Schrödinger equation for the infinite square well, where

$$V(x) = 0 \qquad -a < x < a,$$
$$V(x) = \infty \qquad |x| > a.$$

Hint: you need to use the angle doubling formulas for sines and cosines (such as

$$\sin^2(x) = \tfrac{1}{2}(-\cos 2x + 1),$$

and the primitives of sine and cosine, such as

$$\int \cos(x)dx = \sin(x). \qquad)$$

[15 marks]

2. What is the effect on the energy levels as the depth of the square well potential (a) decreases (b) increases. What happens if the width (c) decreases and (d) increases. Explain your answer. [10 marks]

3. Show that the sum of two solutions to the Schrödinger equation,

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\phi(x) + V(x)\phi(x) = E\phi(x)$$

for two *different* values of the energy is not a solution to the Schrödinger equation. [10 marks]

4. What is the energy for the lowest state of a proton in an infinitely deep square well of a typical nuclear size, 10^{-14} m. The mass of a proton is 1.6725 10^{-27} kg, and $\hbar = 1.0545\ 10^{-34}$ Js. Express the energy in J as well as in MeV, using 1 eV $= 1.6 \cdot 10^{-19}$ J. [10 marks]

5. We consider the asymmetric well in Fig. 1,

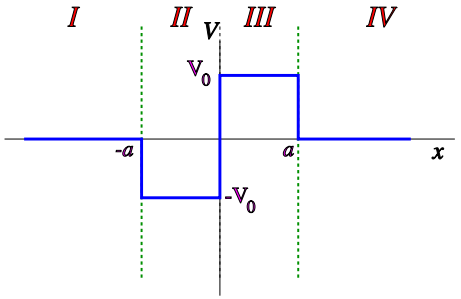

Figure 1: The potential

$$V(x) = \begin{cases} 0 & |x| > a \\ -V_0 & -a < x < 0 \\ V_0 & 0 < x < a \end{cases}.$$

a Write the form of the wave function for $-V_0 < E < 0$ in the regions I, II, III and IV. Define your parameters.

b What are the matching conditions?

c Write the explicit form of these conditions.

A solution of these conditions is *not* required! [15 marks]